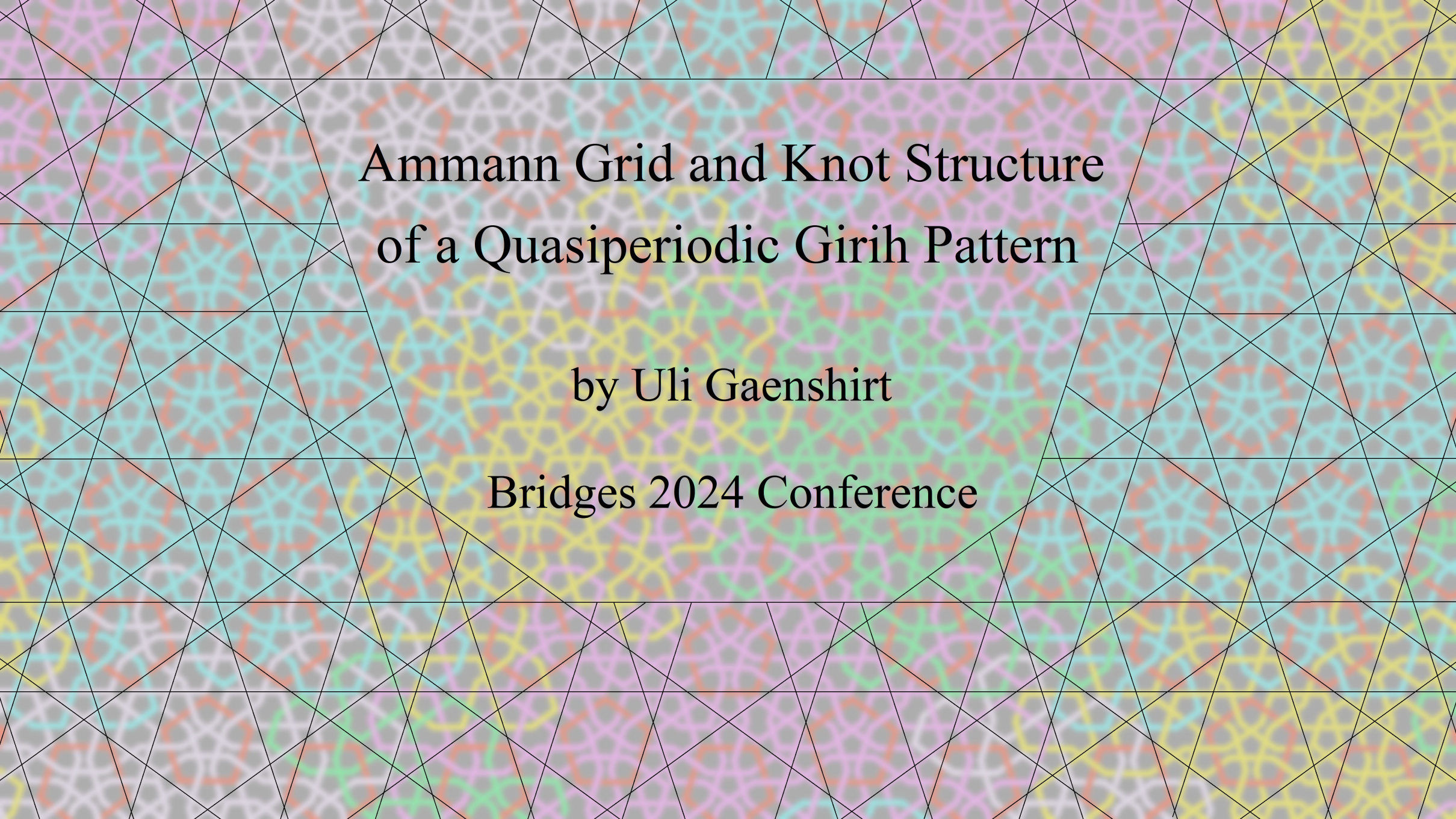


Ammann Grid and Knot Structure of a Quasiperiodic Girih Pattern

by Uli Gaenshirt

Bridges 2024 Conference

Historical Girih pattern

Internal angles are multiples of 36 degrees

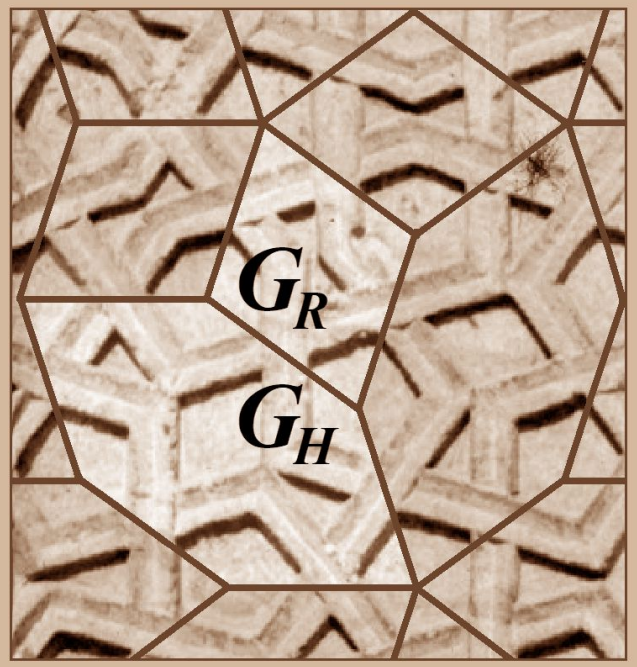
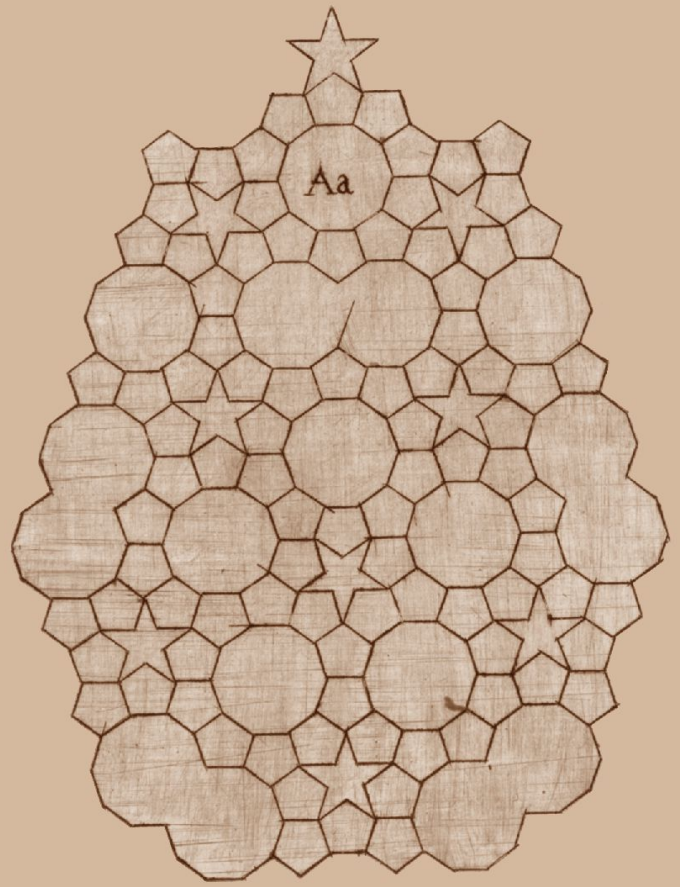


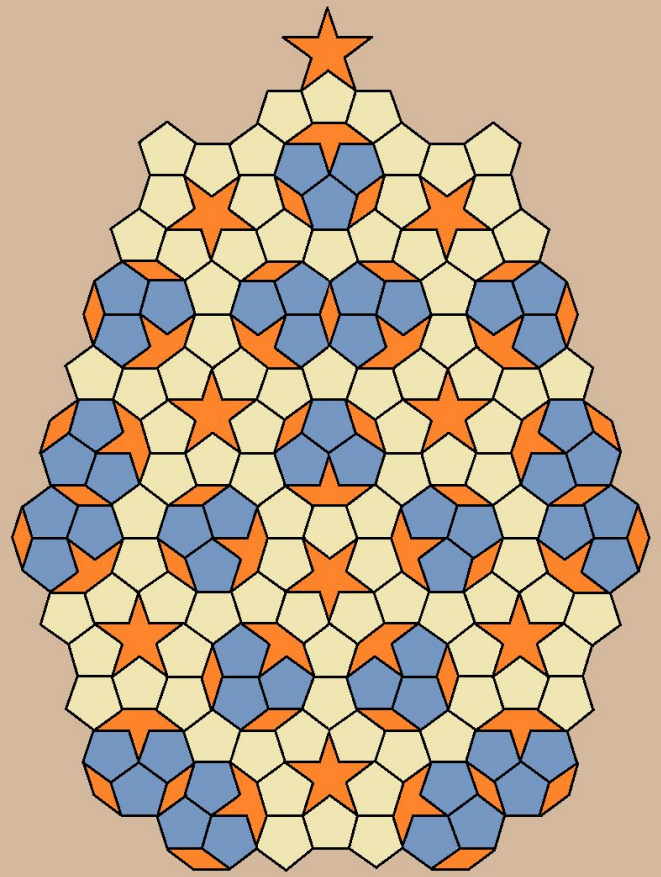
Figure 1(a)

Kepler tiling Aa



(b)

Penrose pentagon tiling



(c)

Penrose pentagon tiling

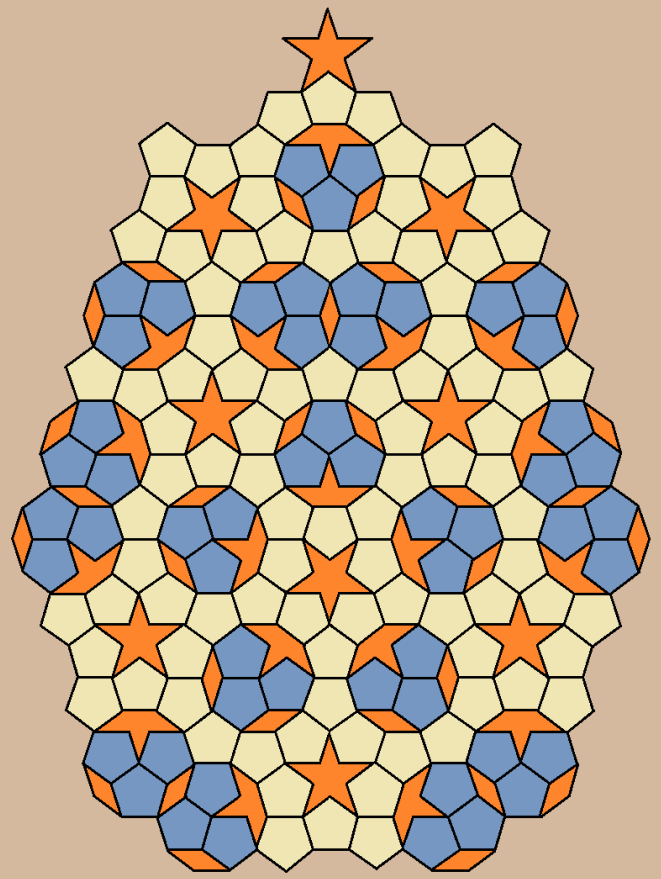
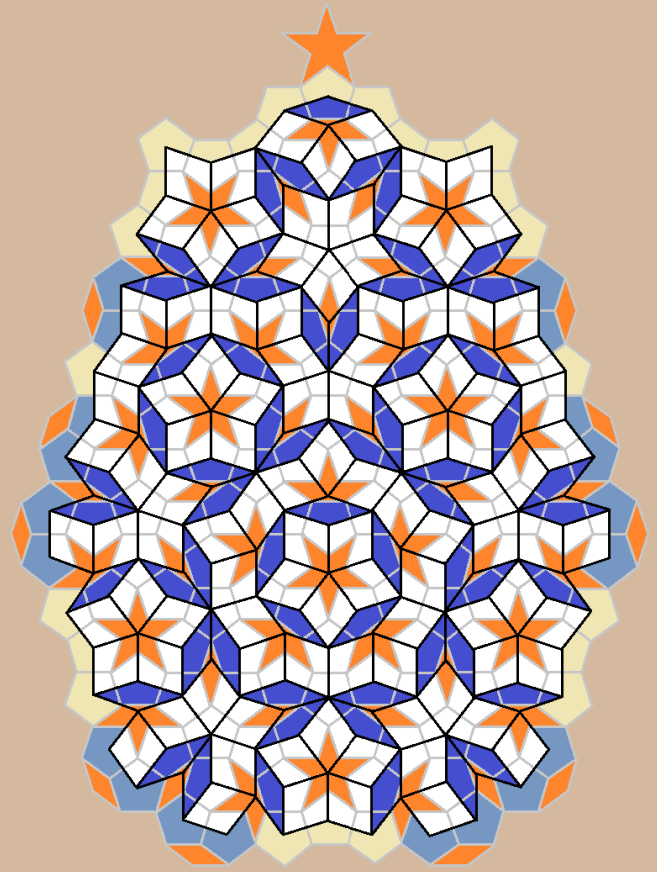


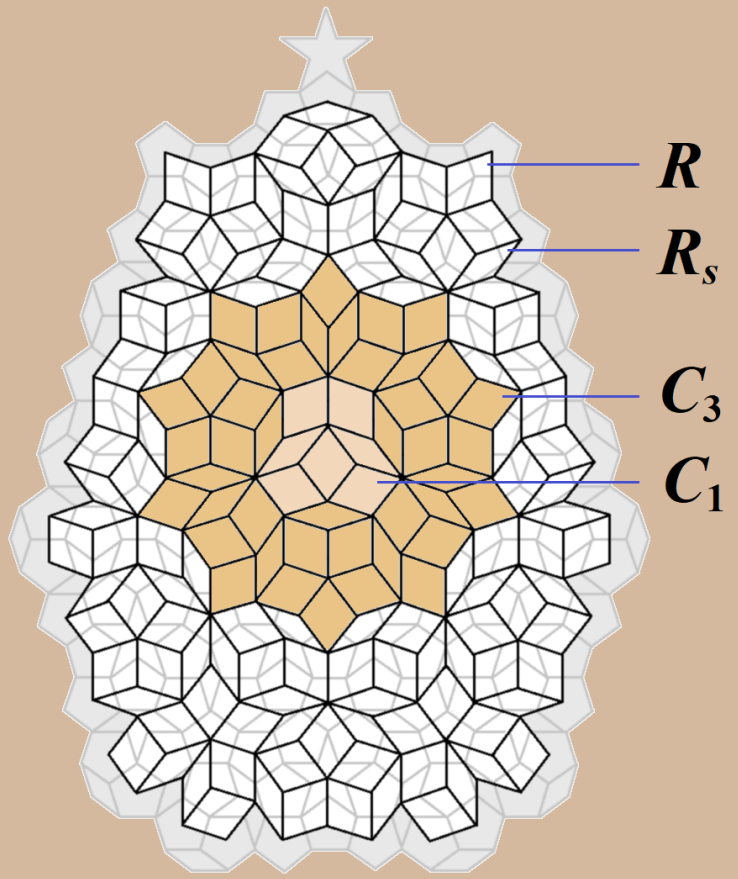
Figure 1(c)

Rhombus tiling derived



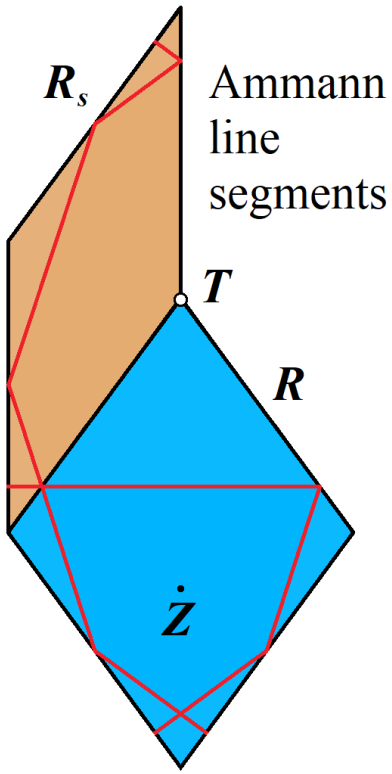
(d)

Rhombus tiling with C_3



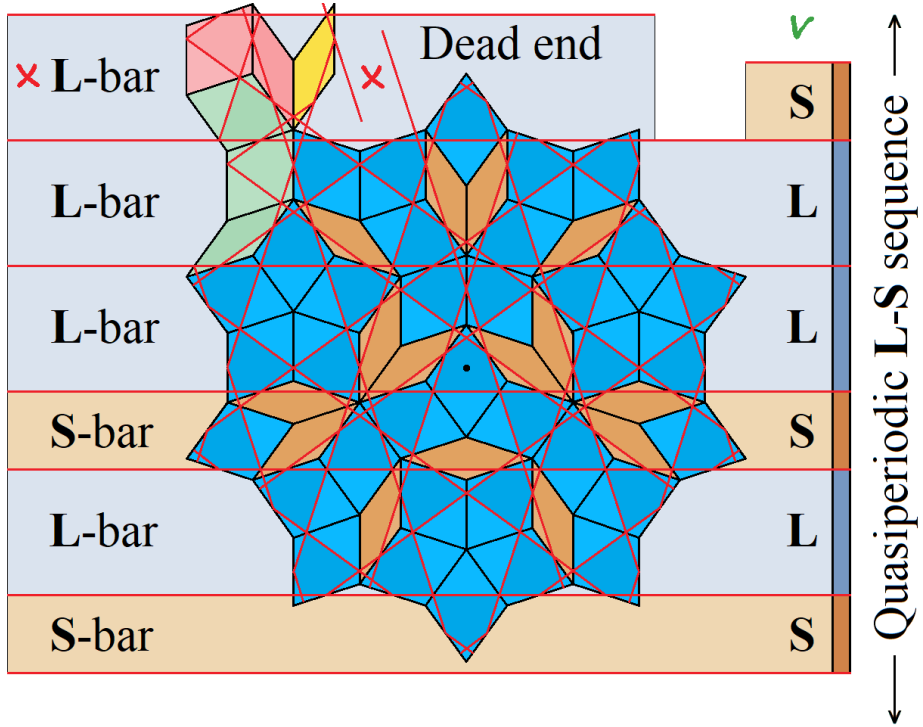
(e)

Matching rules



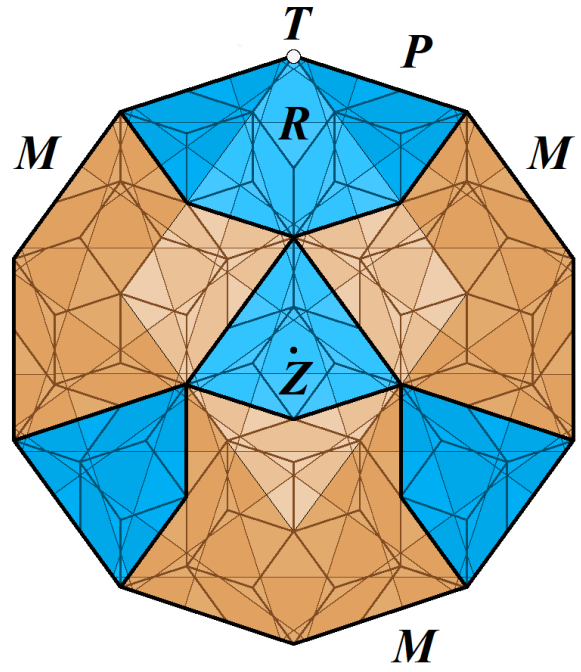
Proto-rhombs

Rule fulfillment in a cartwheel C_3



Continous Ammann lines create a grid
Above: Incompleteness of the rules

Covering rules



Gummelt-Decagon ***P***
(Rule also incomplete)

Figure 2(a) (b) (c)

Substitution of the rhombus R_q and its Ammann lines, shown by the horizontal lines

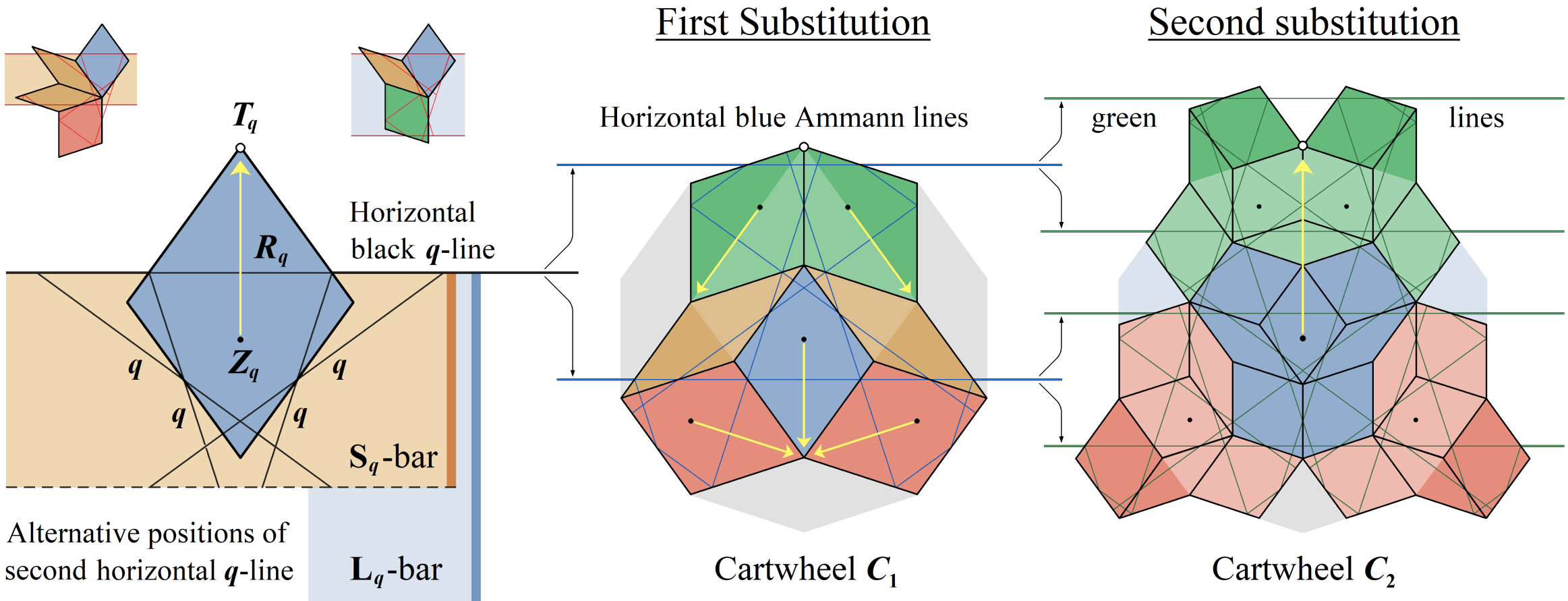


Figure 3(a) (b) (c)

Third substitution

Ammann grid of the quasi-cell $\boldsymbol{Q}$

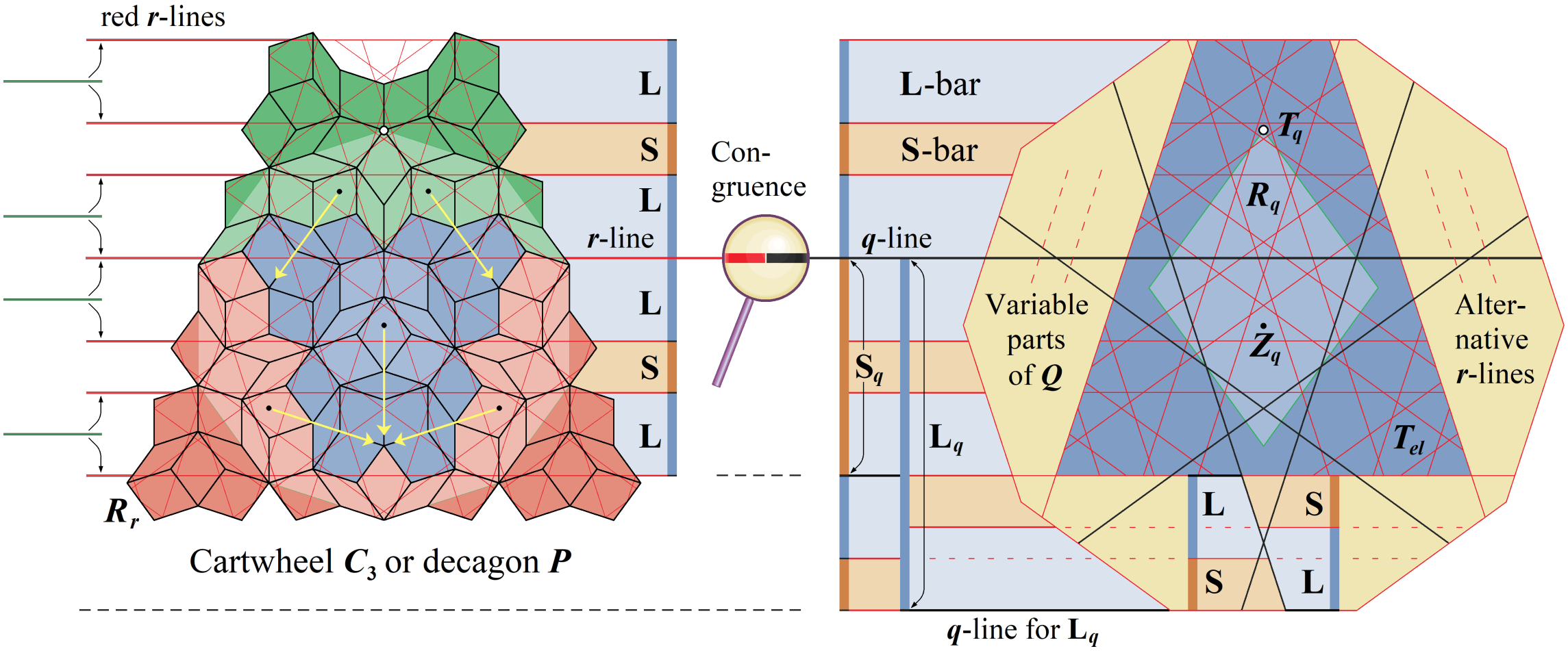


Figure 3(d)

(e)

Girih pattern with highlighted Girih stencils

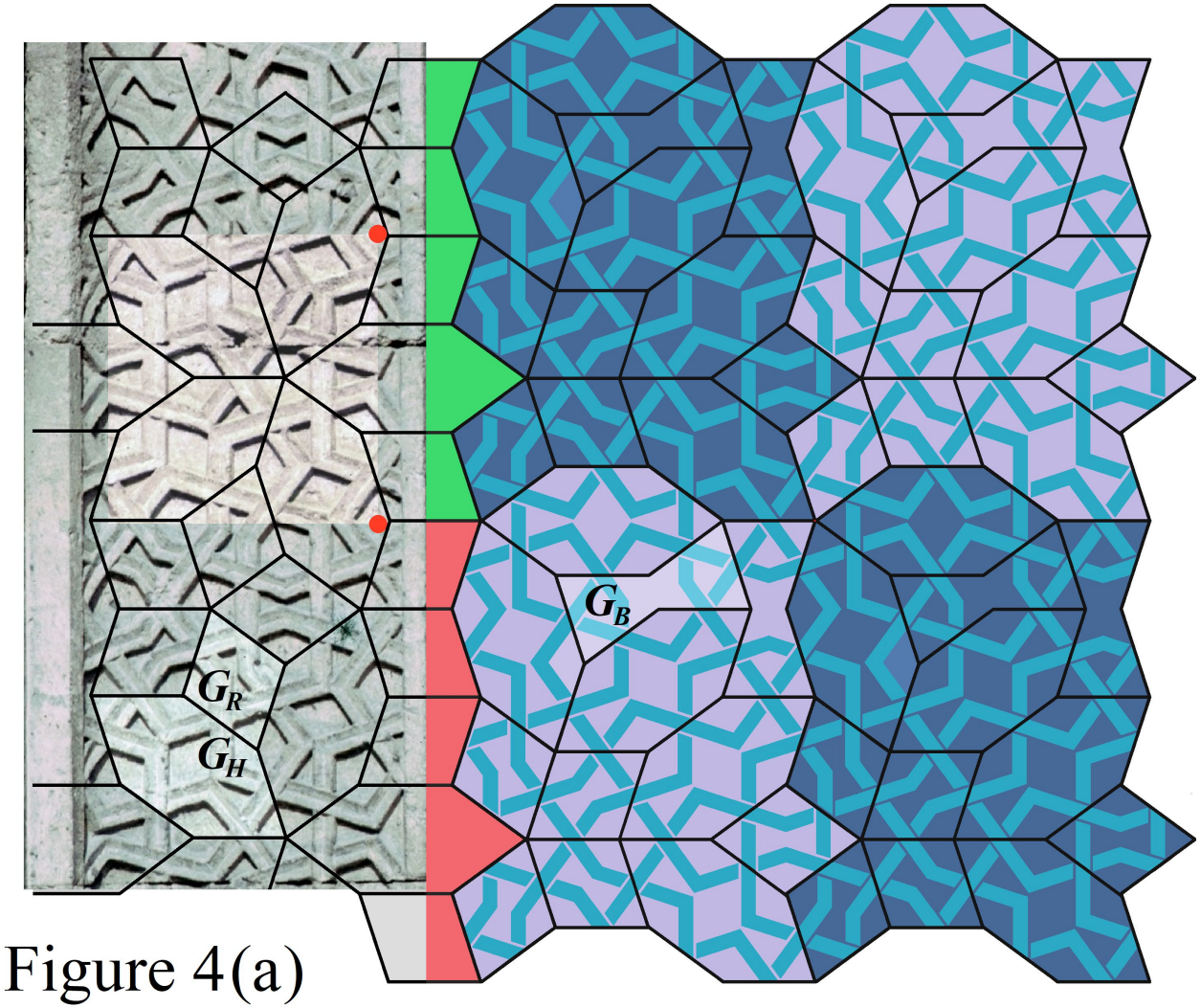
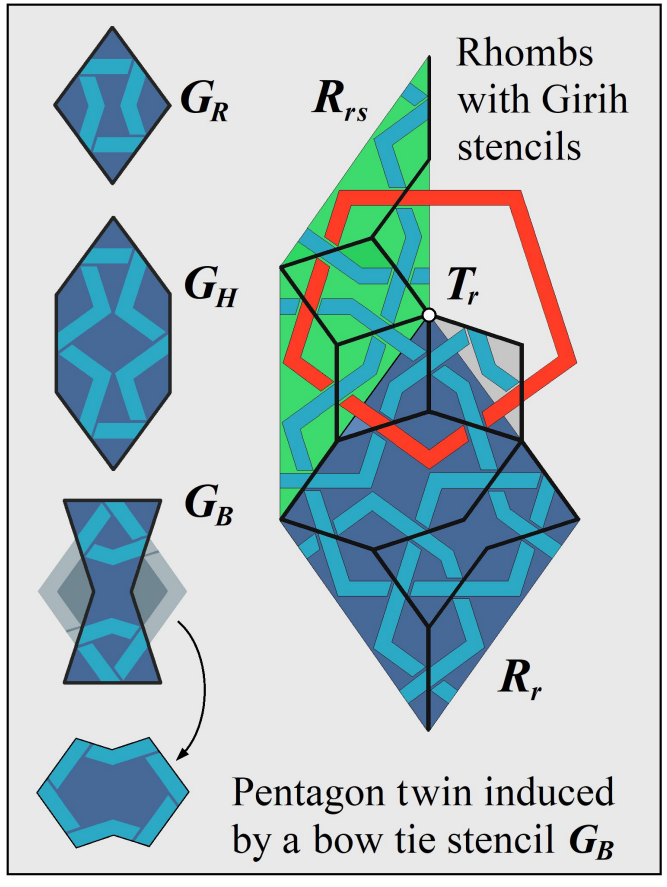


Figure 4(a)

Incorporation into rhombs



(b)

Incorporated stencils

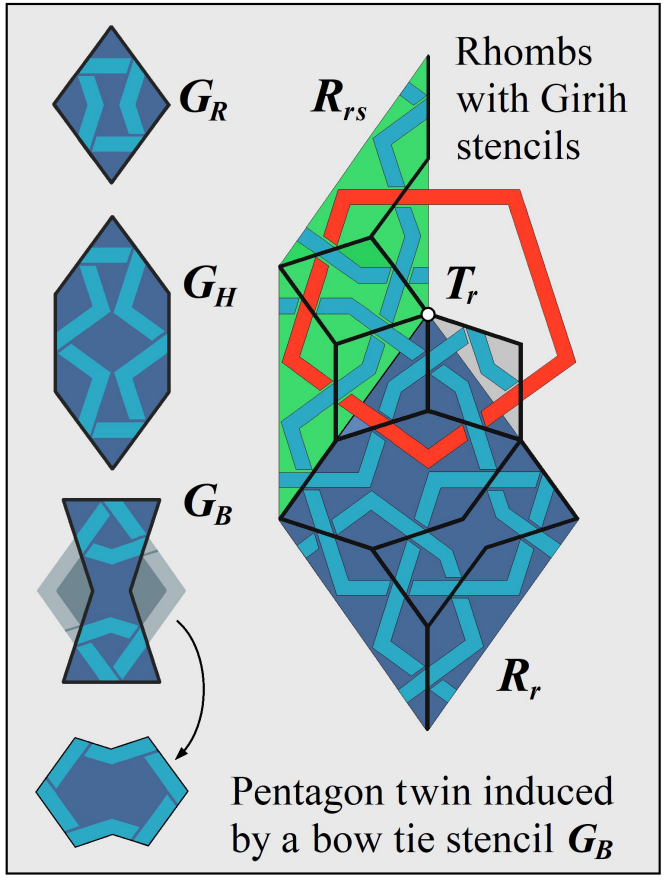
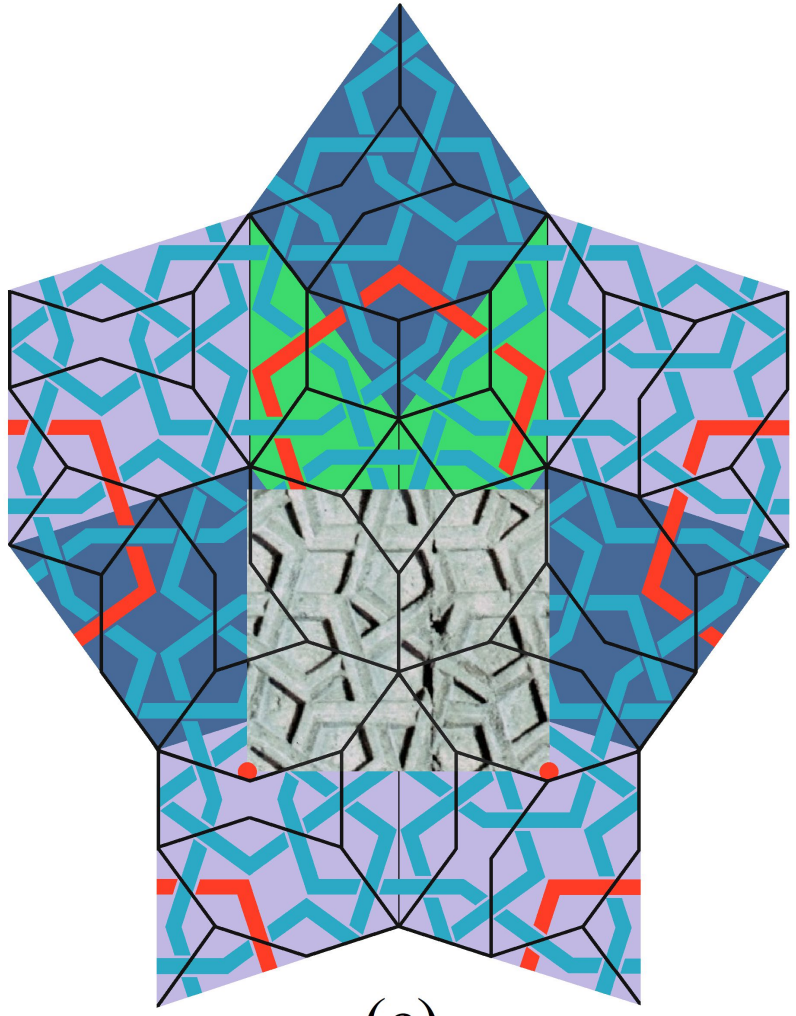


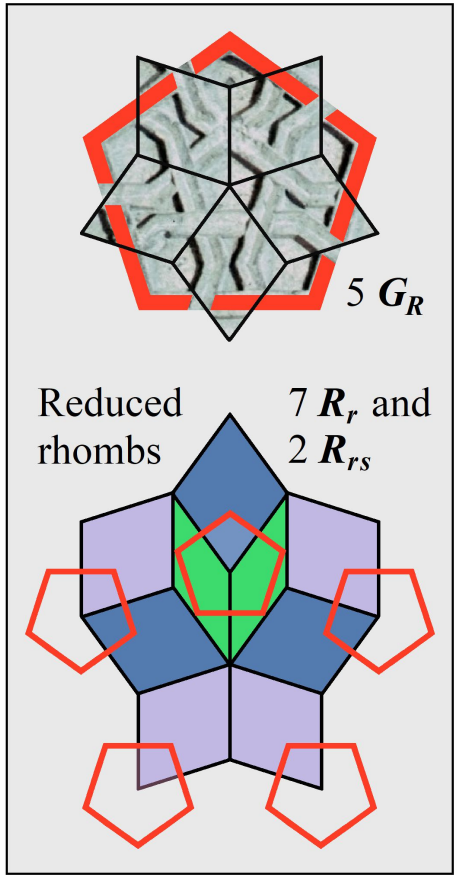
Figure 4(b)

Quasiperiodic Girih pattern



(c)

Girih pentagons



(d)

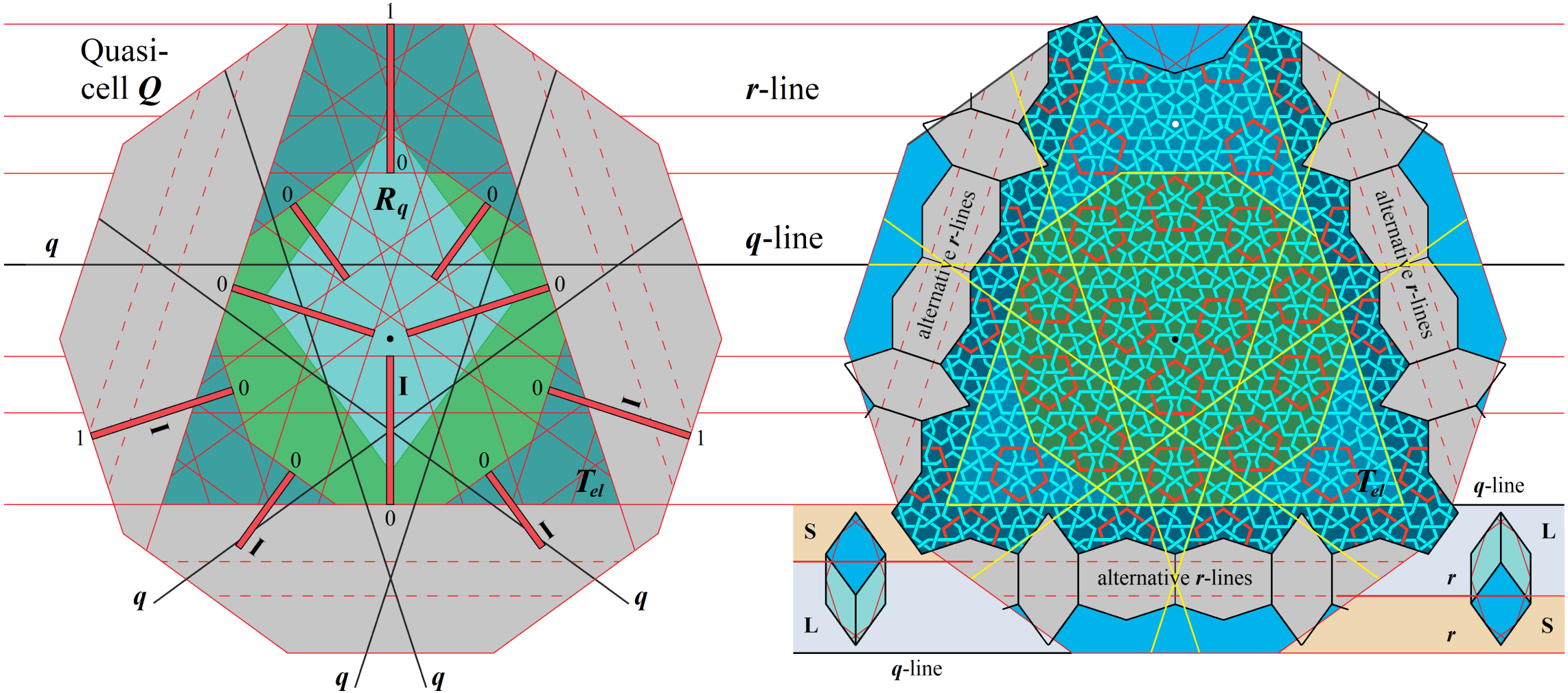
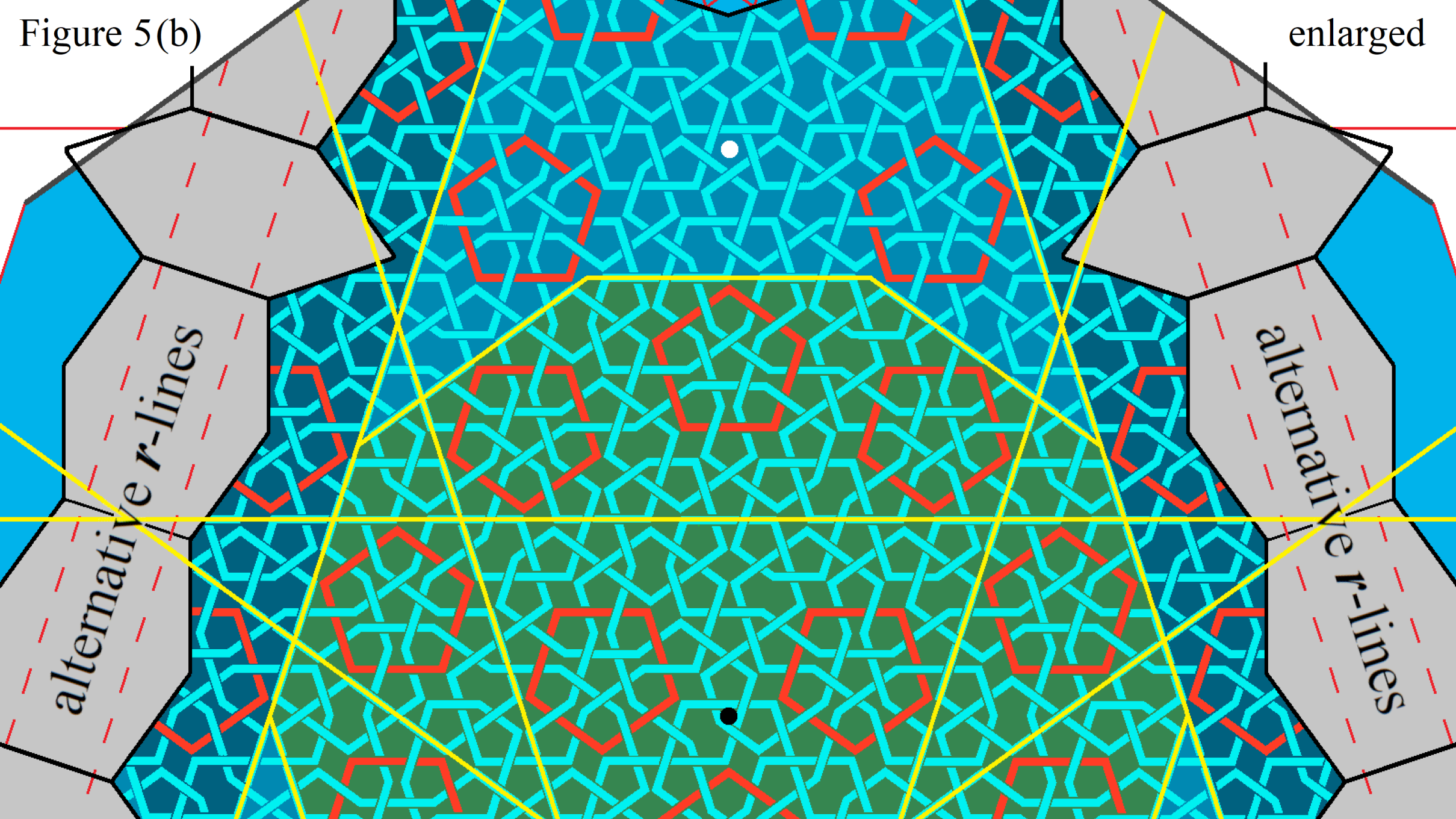

Q with twin-scales I and q-lines
Q with Girih pattern and flipping worms
Quasi-cell Q
r-line
q-line
q
R_q
I
T_el
alternative r-lines
alternative r-lines
alternative r-lines
S
L
L
S
r
r
q-line
q-line
0
1

Figure 5 (a) (b)

Figure 5(b) enlarged



Q with specified lower flip type

Three specified flips & three Girih knot types

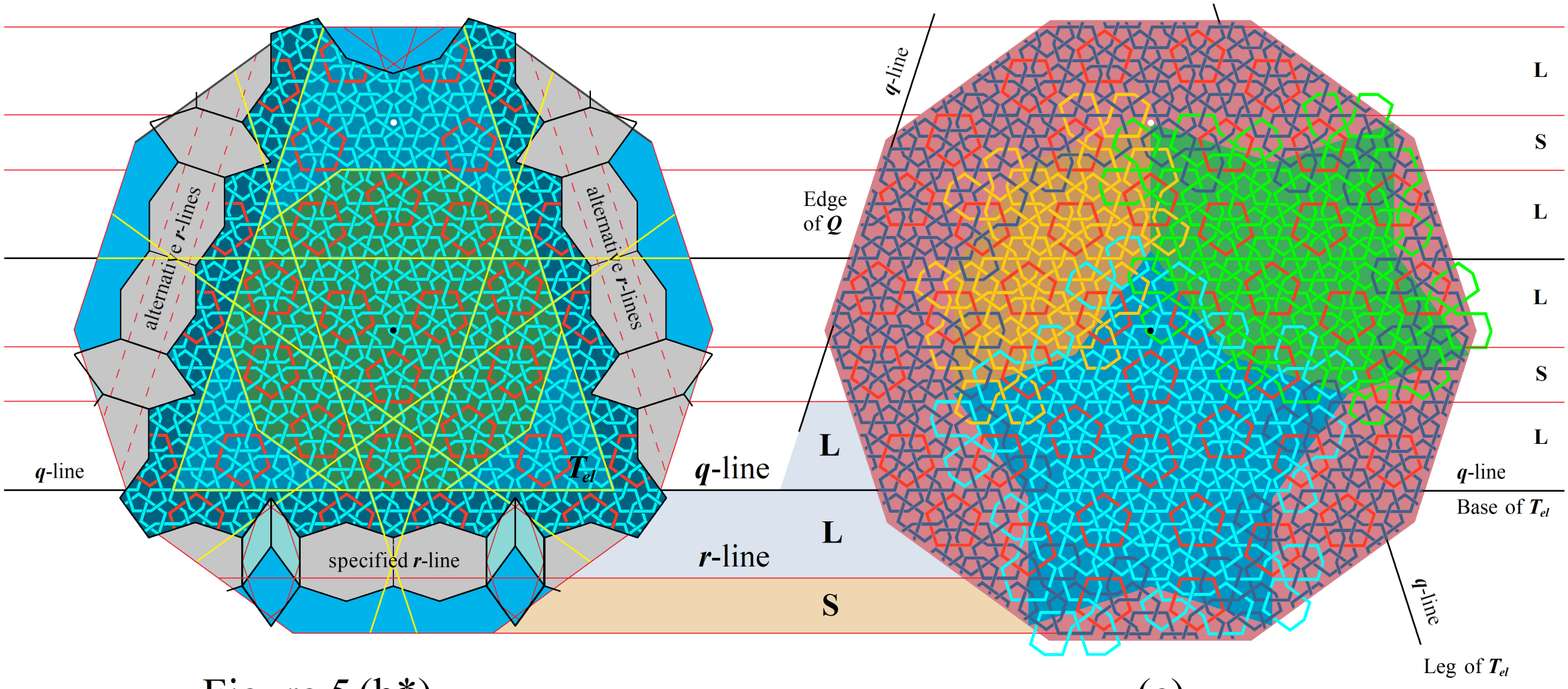
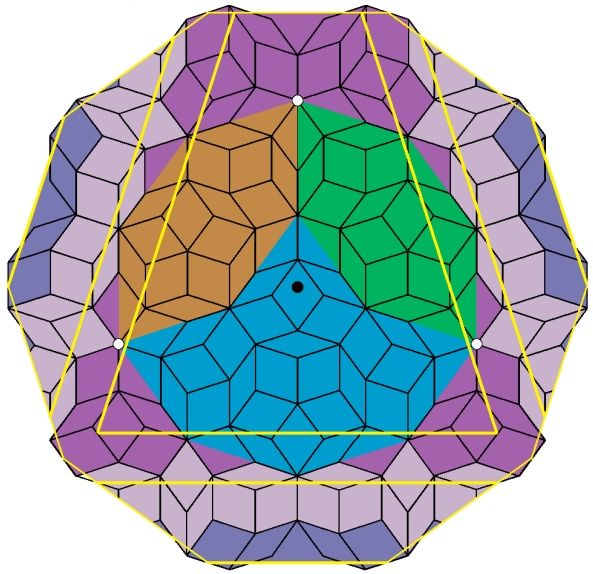


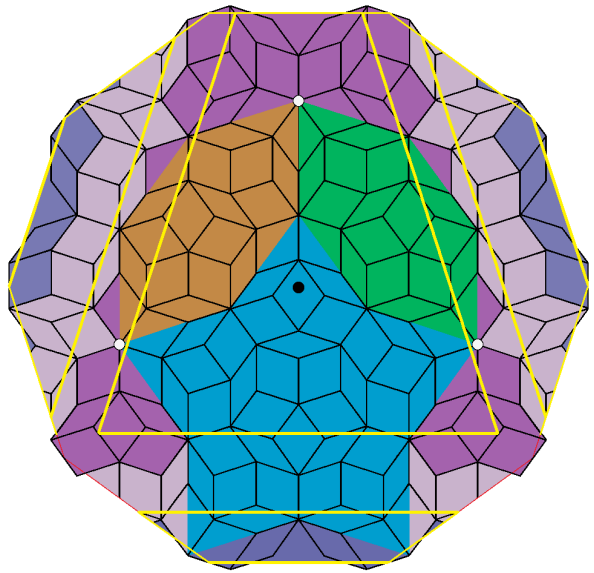
Figure 5 (b*)

(c)

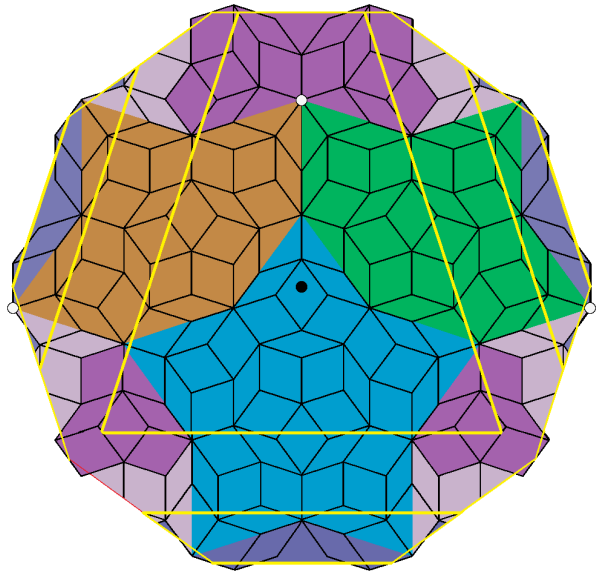
H-B-H



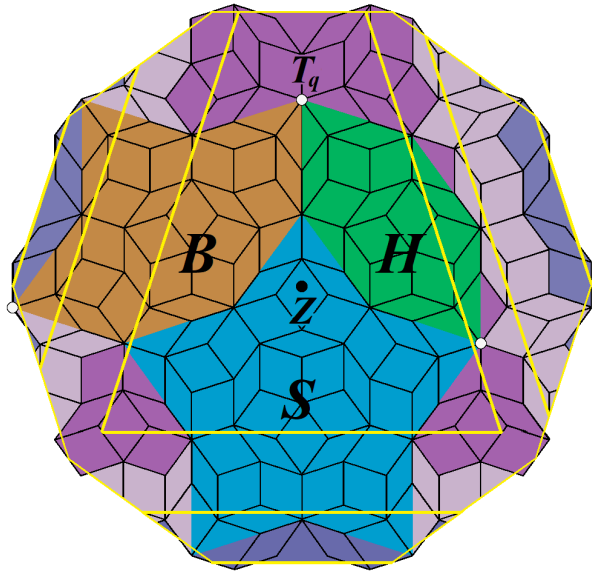
H-S-H



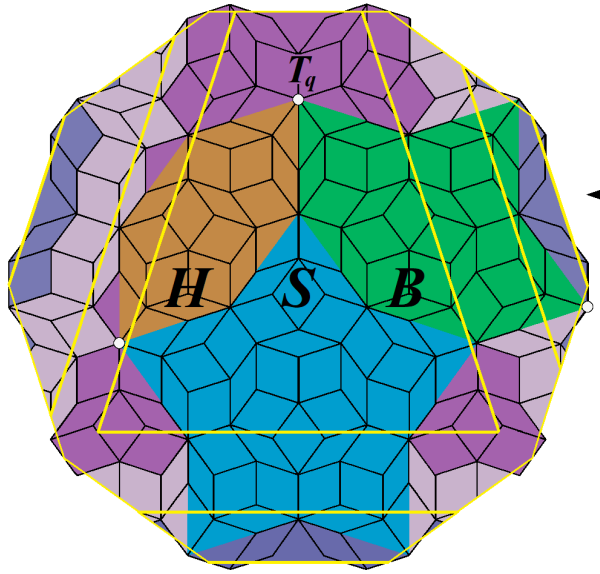
B-S-B

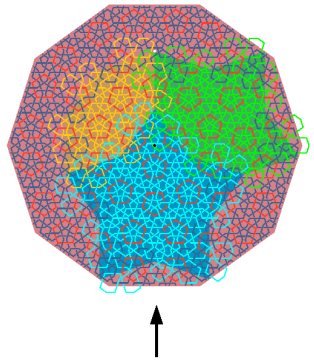


B-S-H



H-S-B

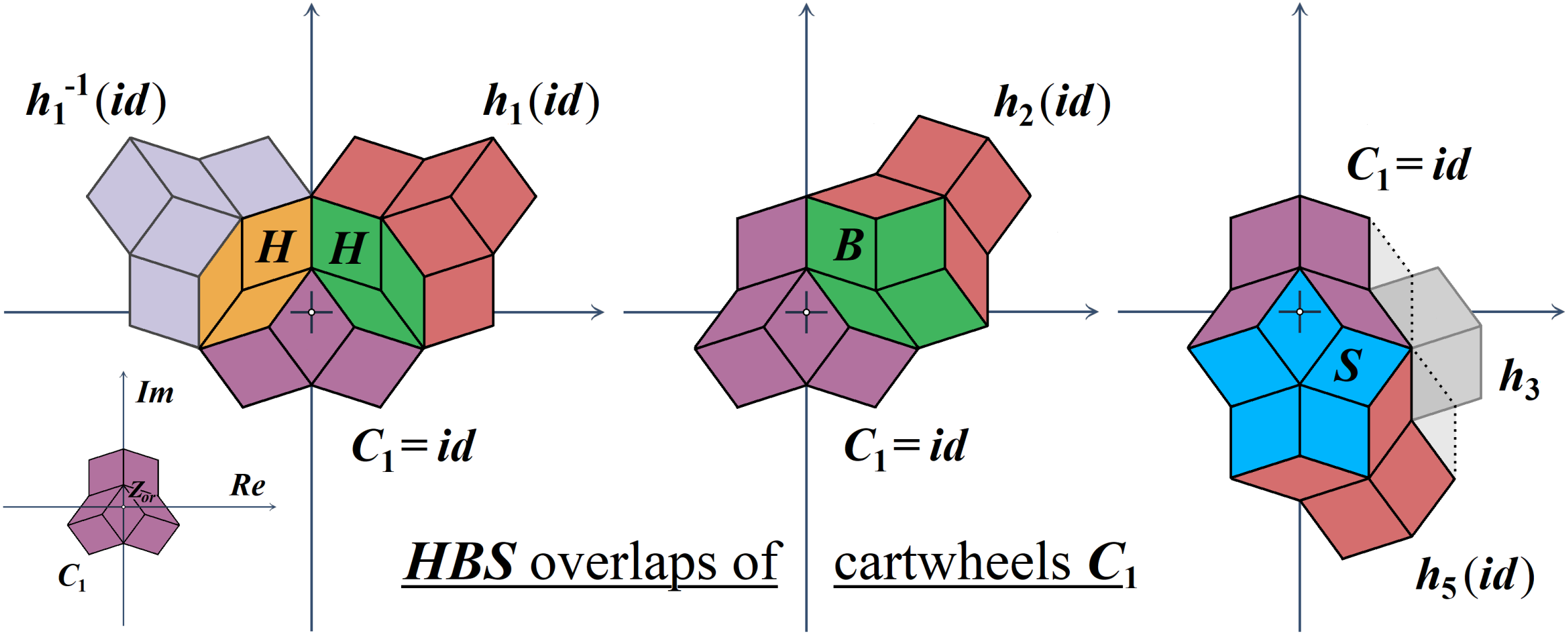




The five status types of the quasi-cell ***Q***

Figure 5(d)

Figure 5(c)



HBS overlaps of cartwheels C_1

Hexagon tile overlaps

Boat tile overlap

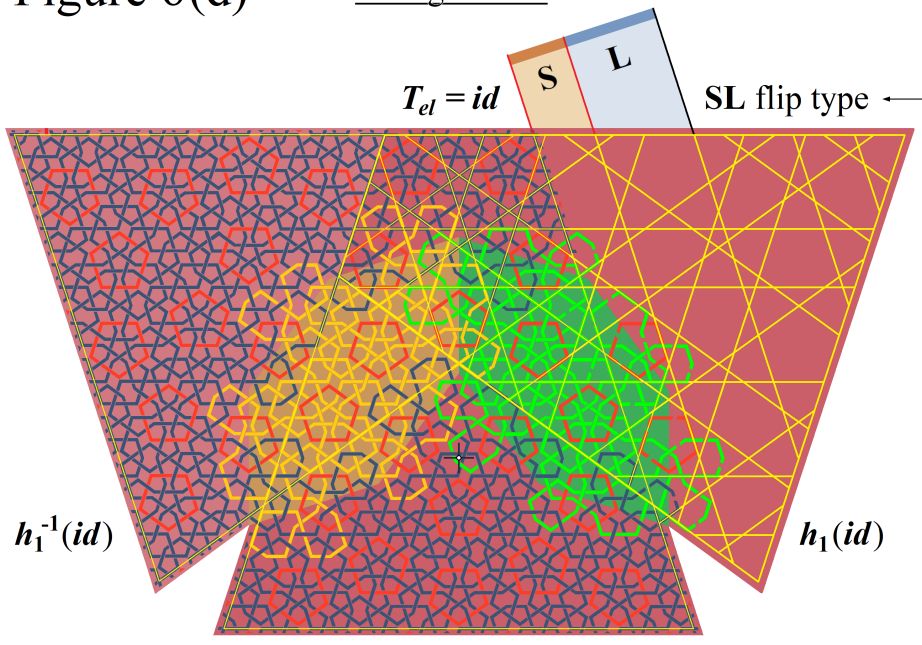
Star tile overlap

Figure 6(a)

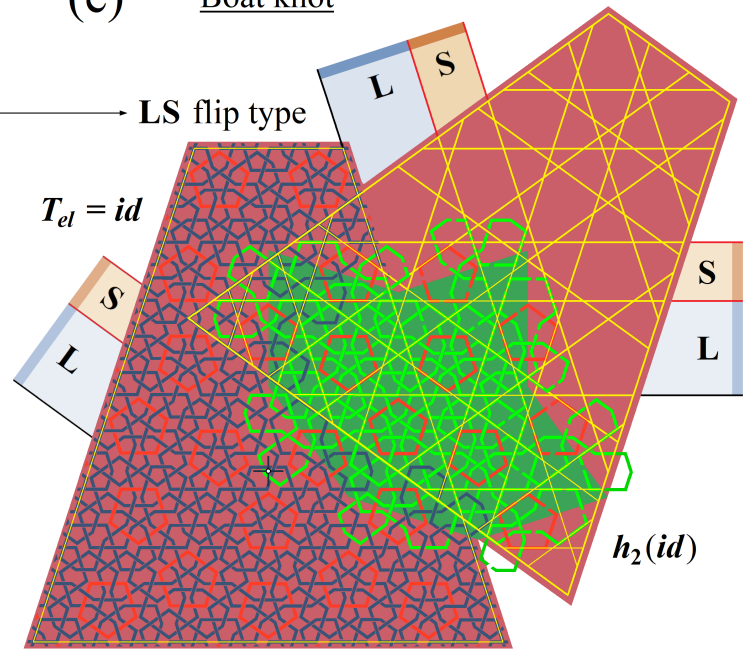
(b)

(c)

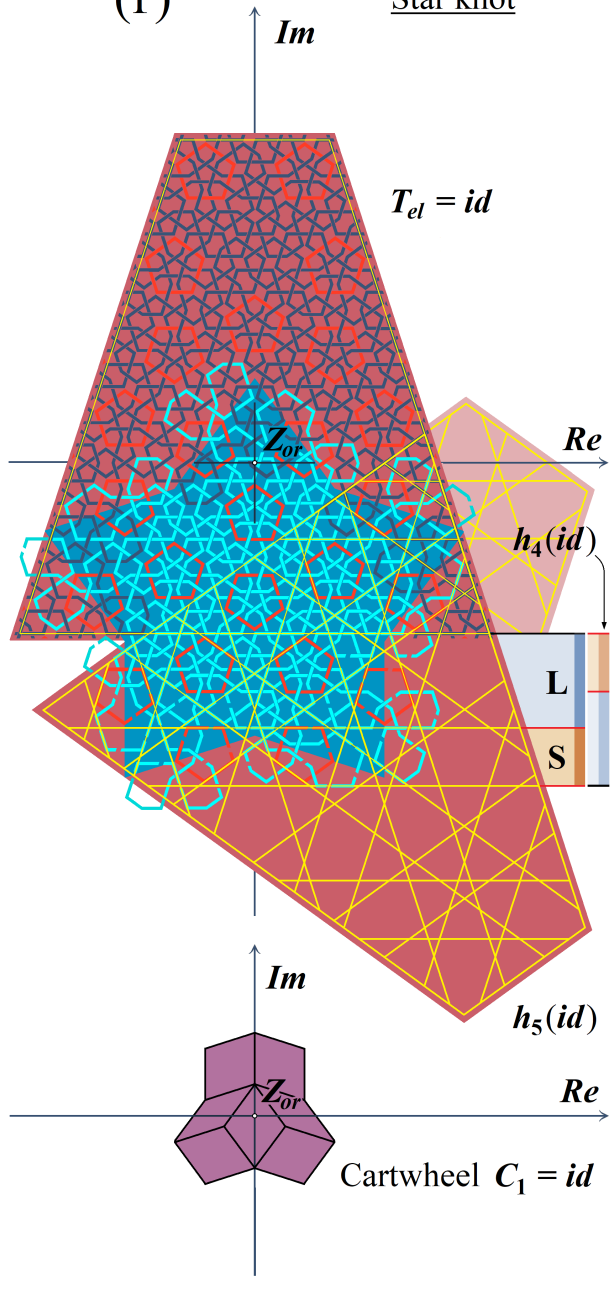
Figure 6(d) Hexagon knot



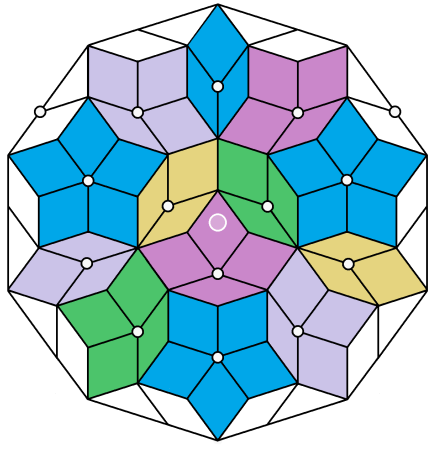
(e) Boat knot



(f) Star knot

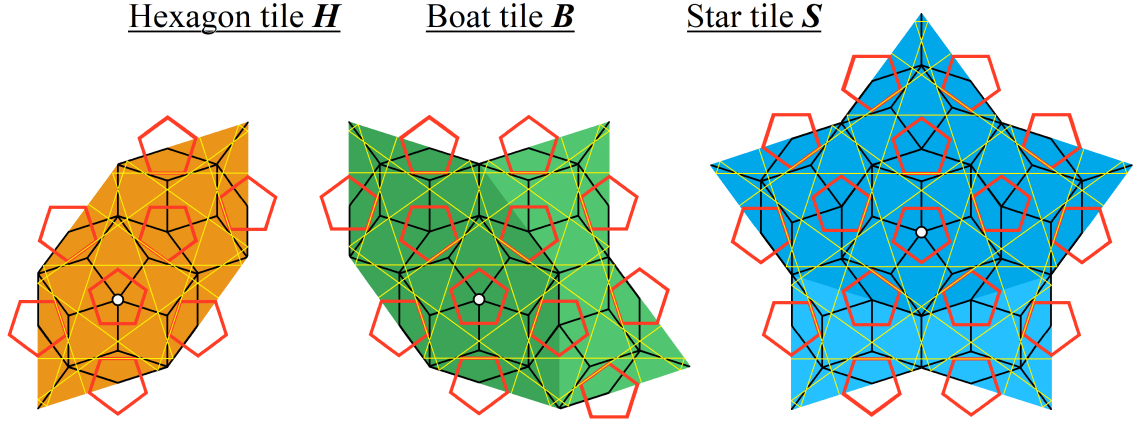


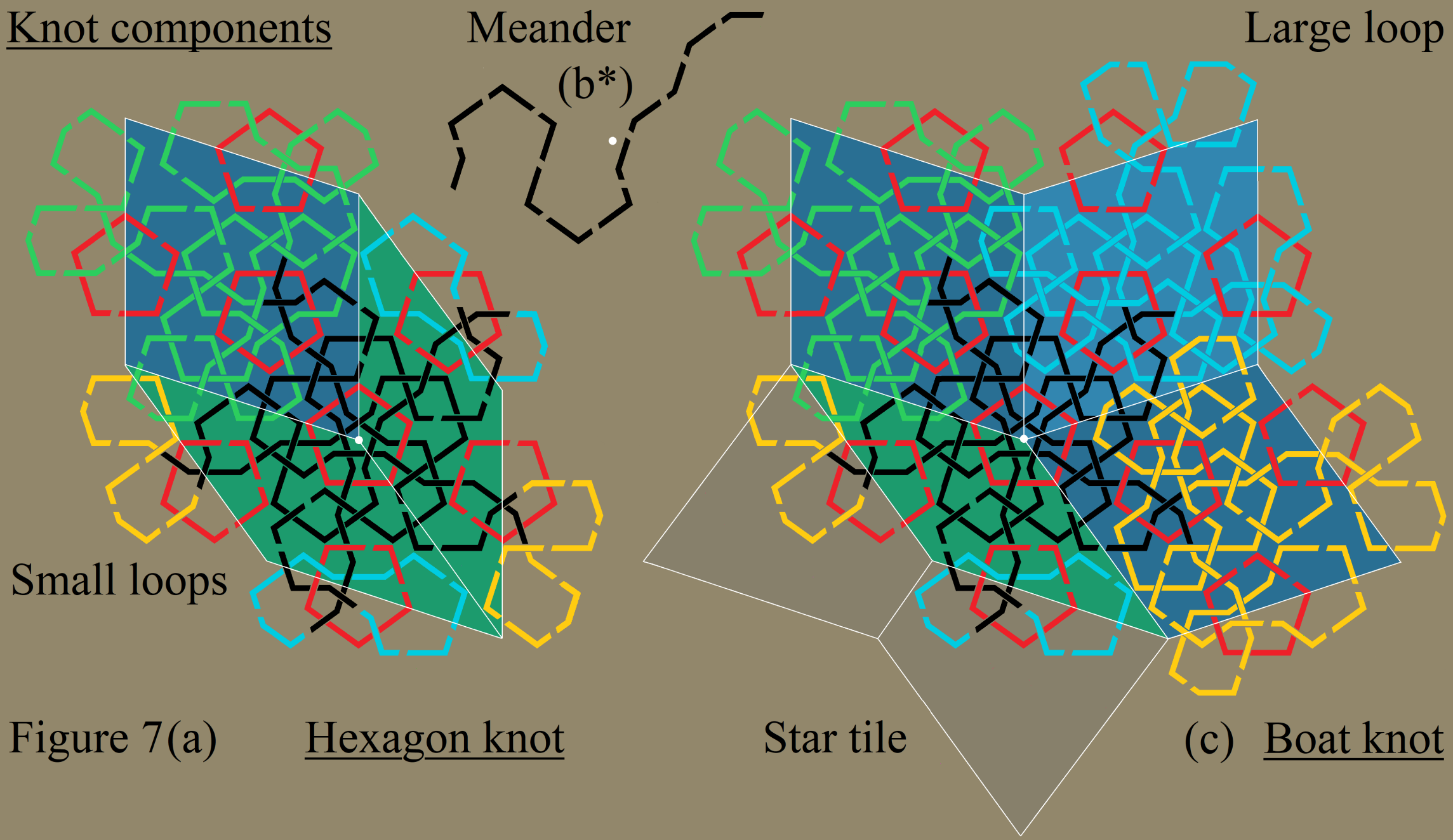
(g) ***HBS*** tiles in cartwheel order

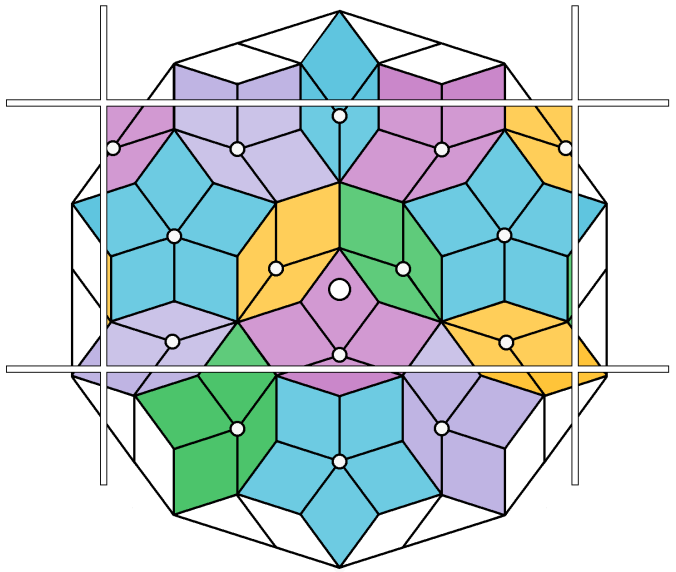


(h) ***HBS*** tiles with R_r rhombus structure, red Girih pentagons and subgrid

Hexagon tile ***H*** Boat tile ***B*** Star tile ***S***



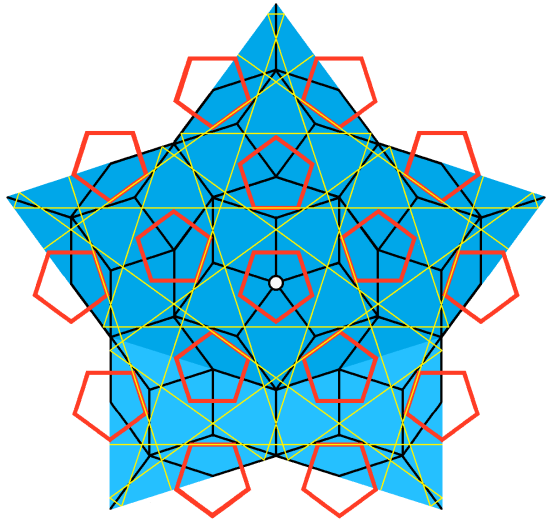

Knot components
Meander
(b*)
Large loop
Small loops
Figure 7(a)
Hexagon knot
Star tile
(c)
Boat knot

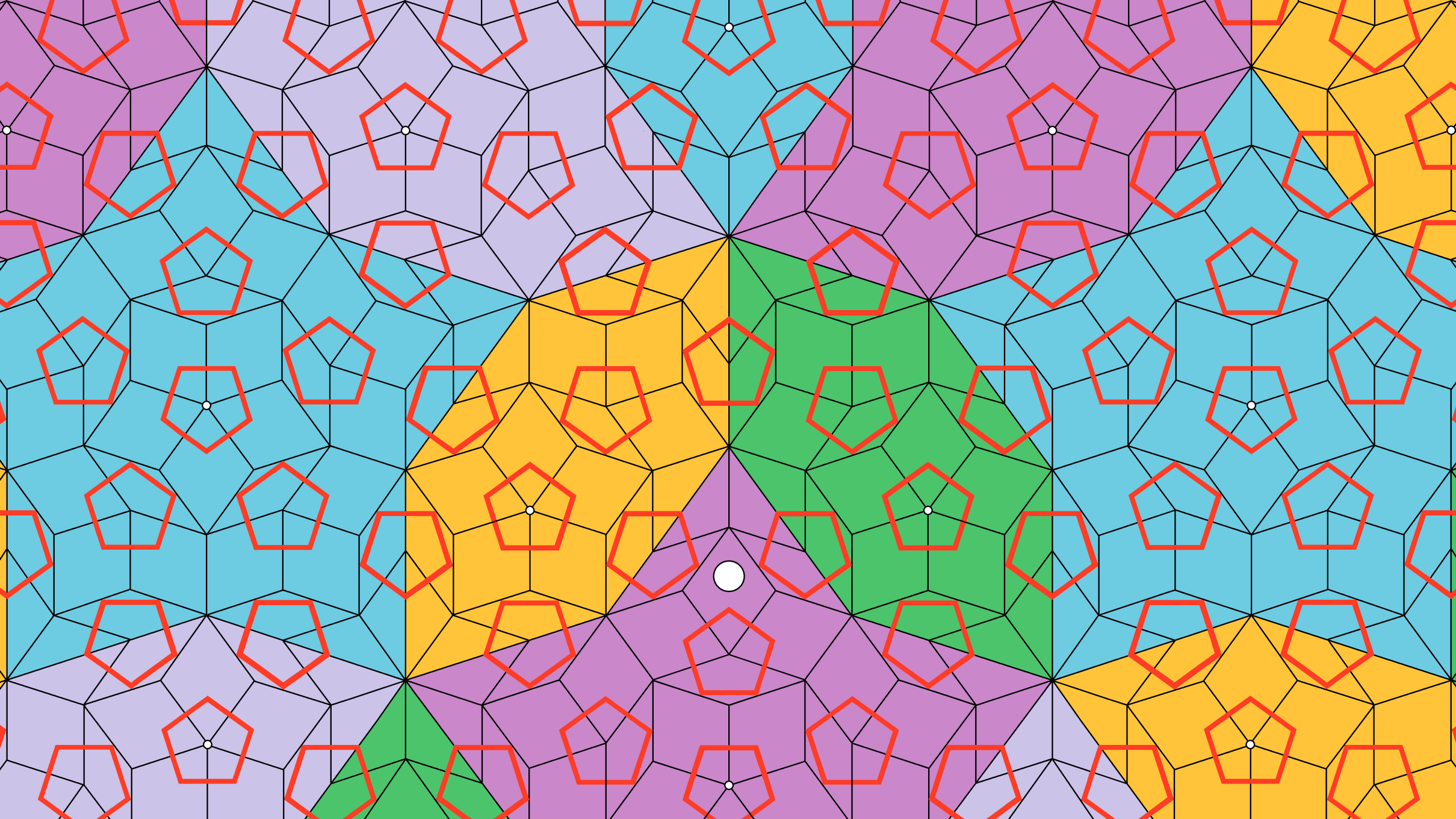


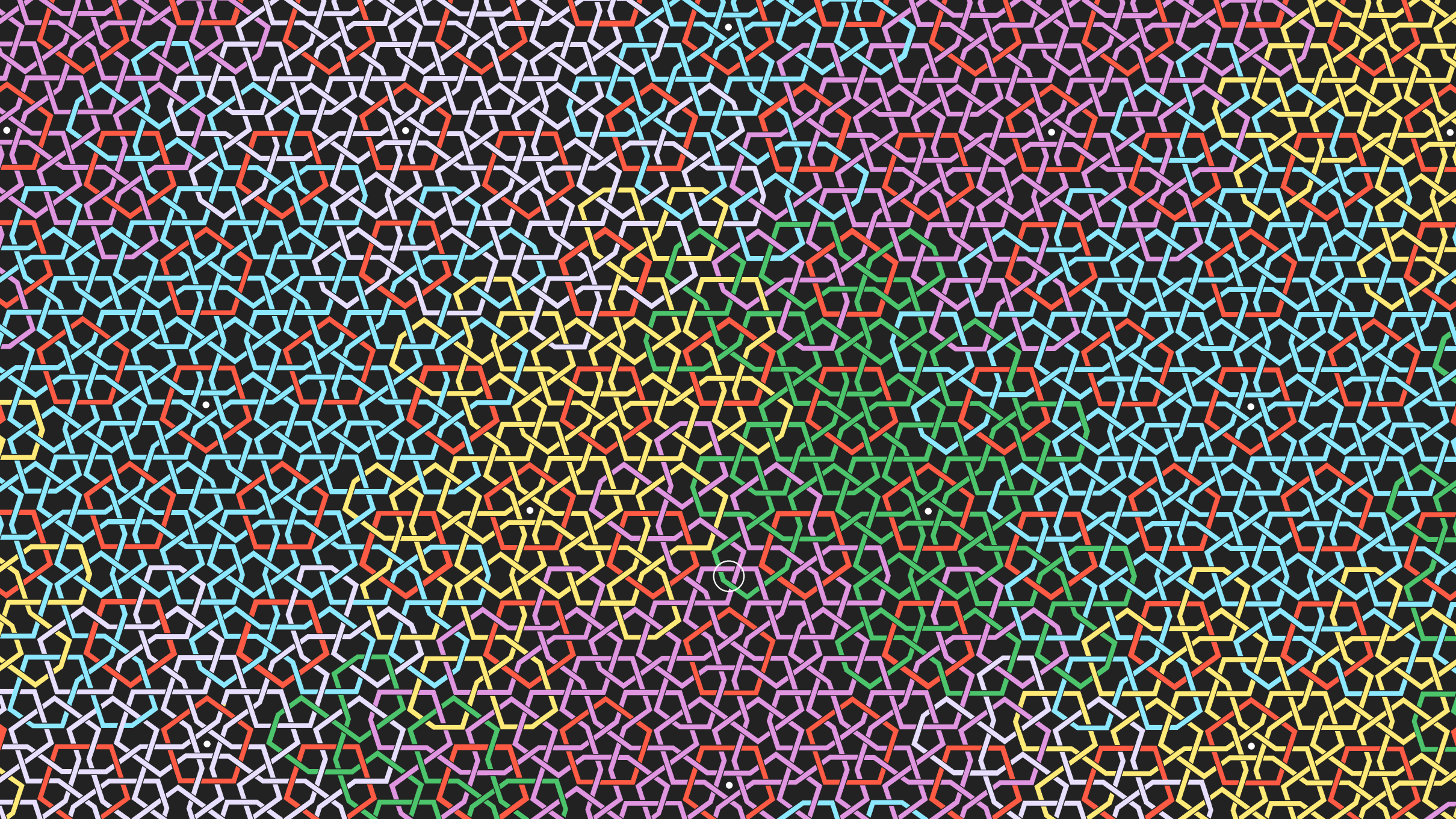
(g)

Figure 6
(g)
enlarged

(h)

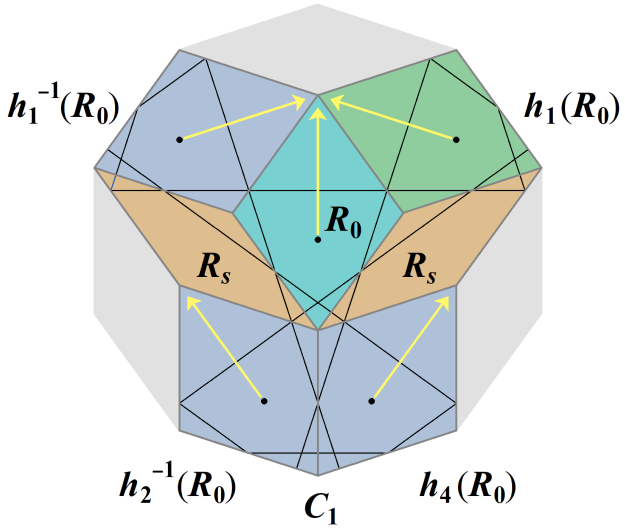






Admissible and forbidden transformations controlled by the quasiperiodic succession algorithm

Each rhombus tiling which is generated by the succession algorithm consists on cartwheels nested inside each other with a start rhombus R_0 in the center of C_1.

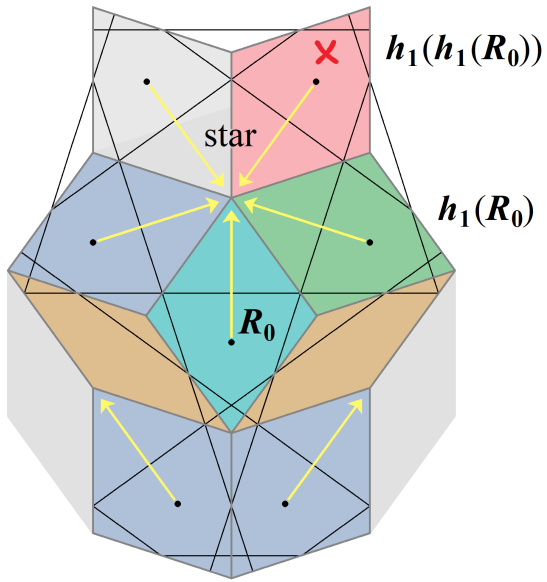


Fulfillment of:

Matching rules: ✓
Succession rule: ✓

Figure A(a)

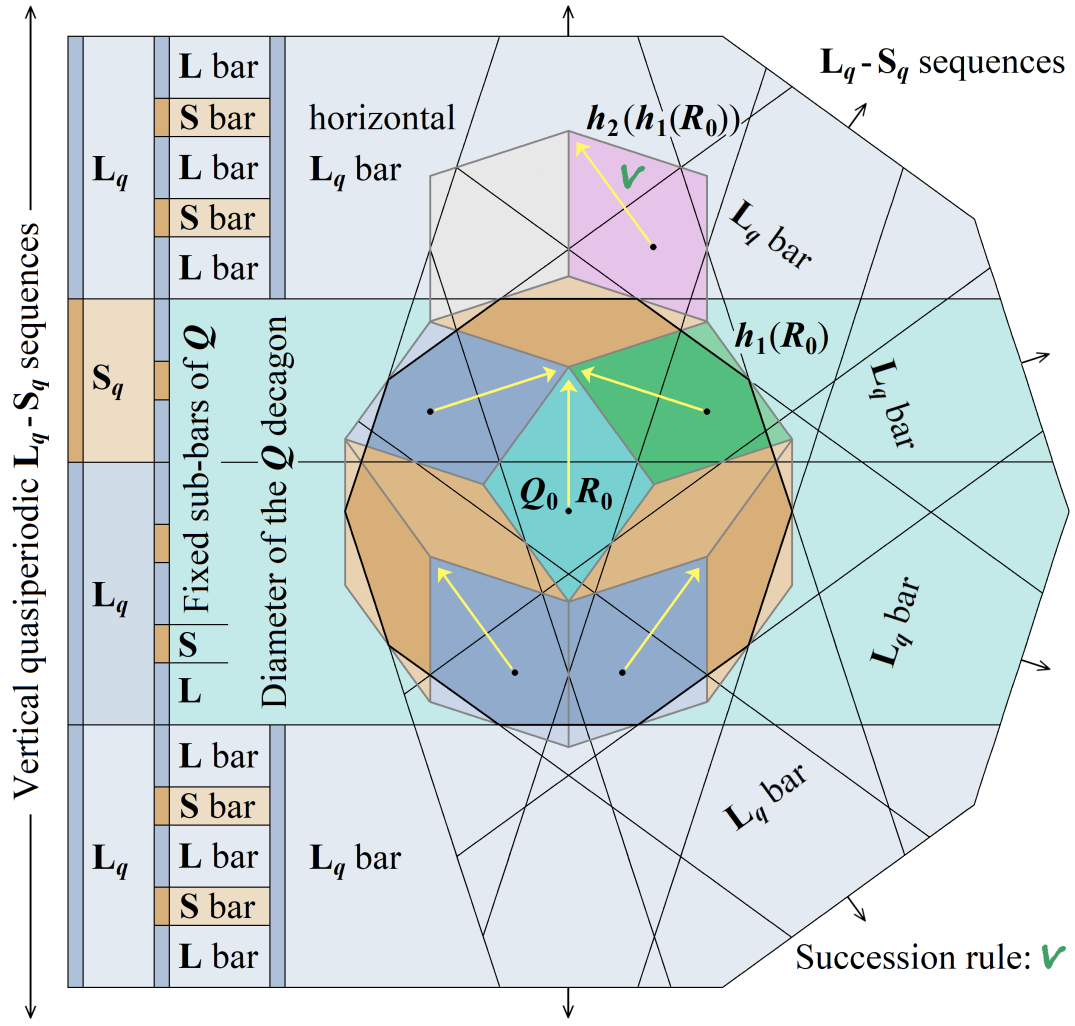
Therefore R_0 cannot belong to a rhombus star as shown below.



Fulfillment of:

Matching rules: ✓
Succession rule: ✗

(b)



(c)

Scale value correlation of overlapping quasi-cells Q shown by the transformation $h_1(Q)$

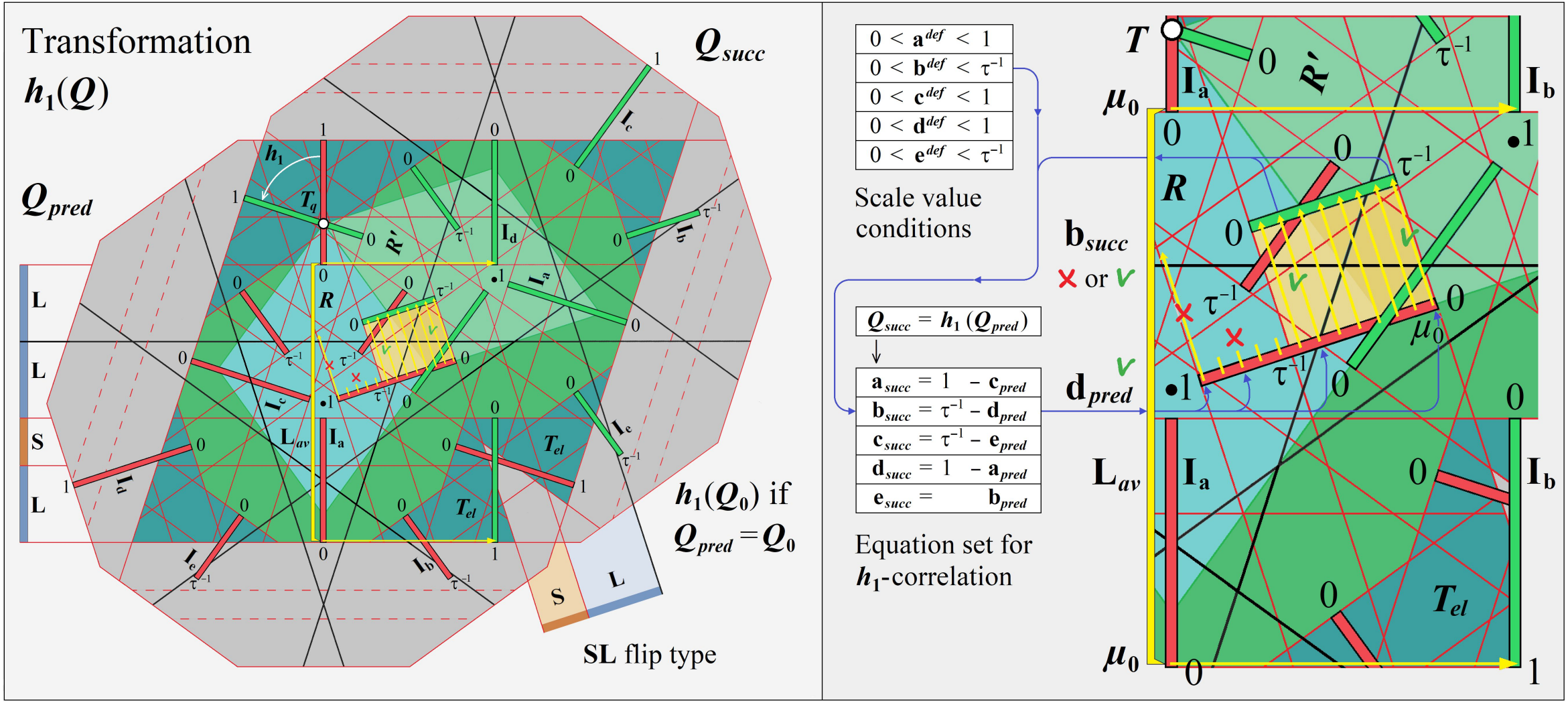


Figure B(a) (b)