

**Ammann Grid and Knot Structure  
of a Quasiperiodic Girih Pattern**

**by Uli Gaenshirt**

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## Historical Girih pattern

Internal angles are  
multiples of 36 degrees

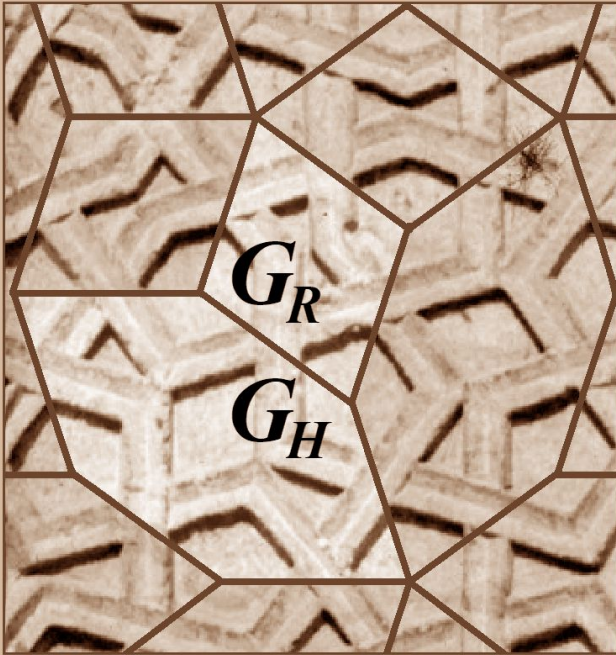
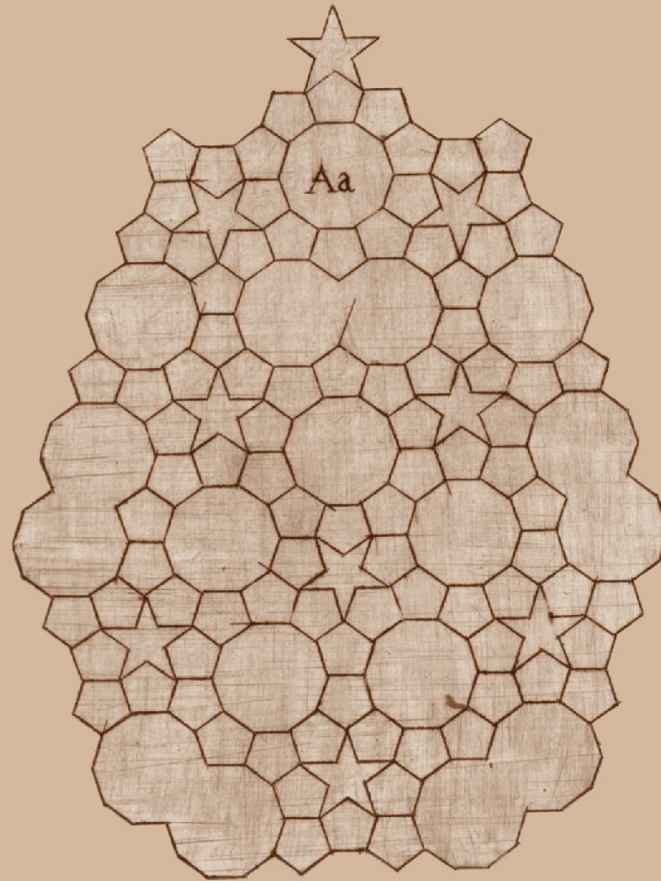


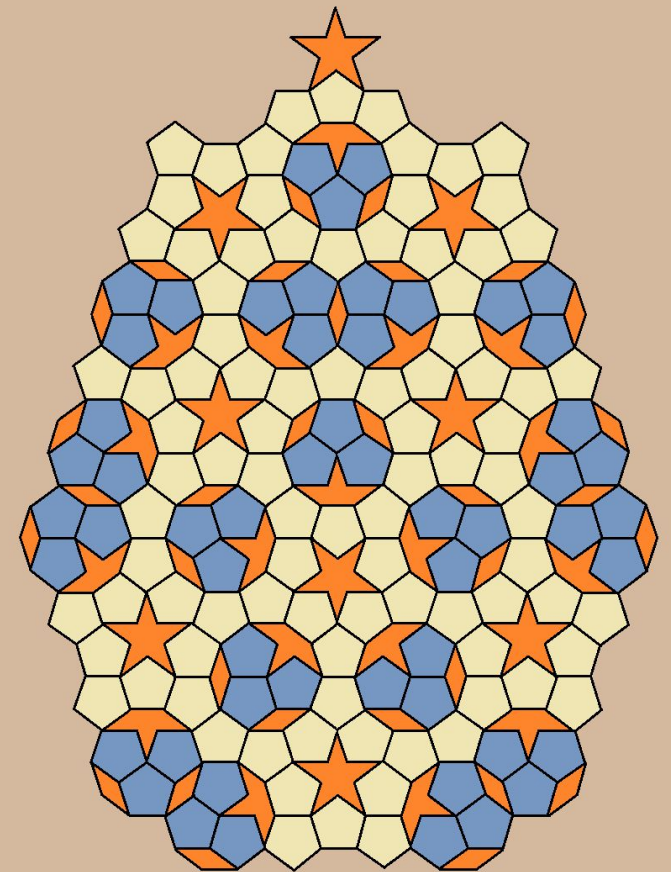
Figure 1(a)

## Kepler tiling Aa



(b)

## Penrose pentagon tiling



(c)

Penrose pentagon tiling

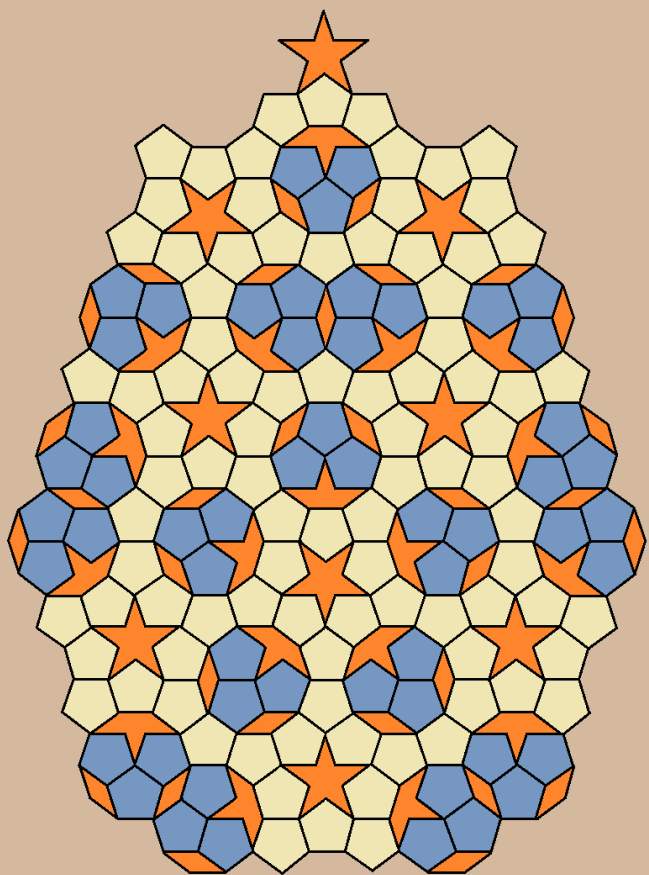
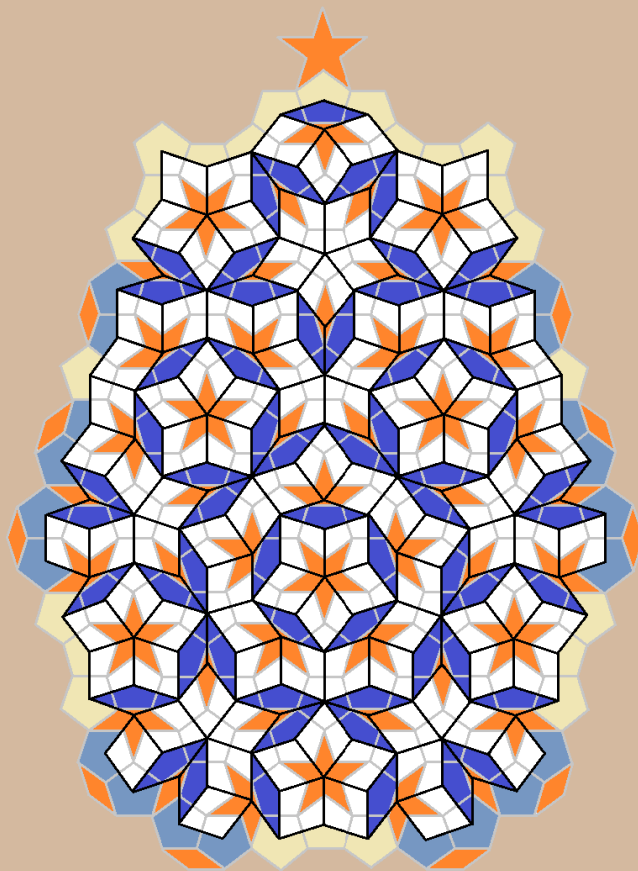


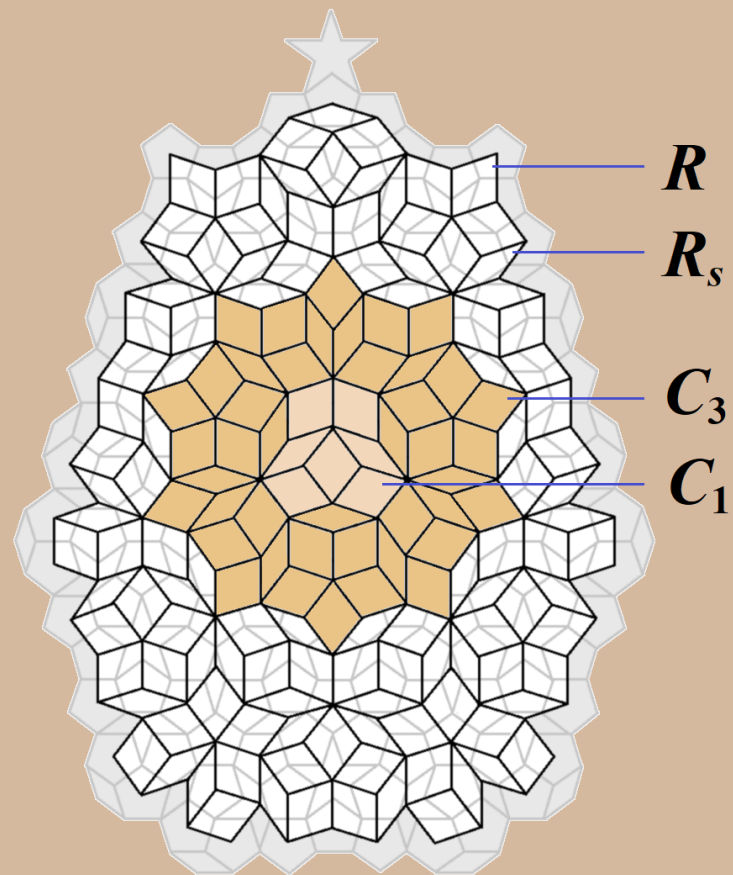
Figure 1(c)

Rhombus tiling derived



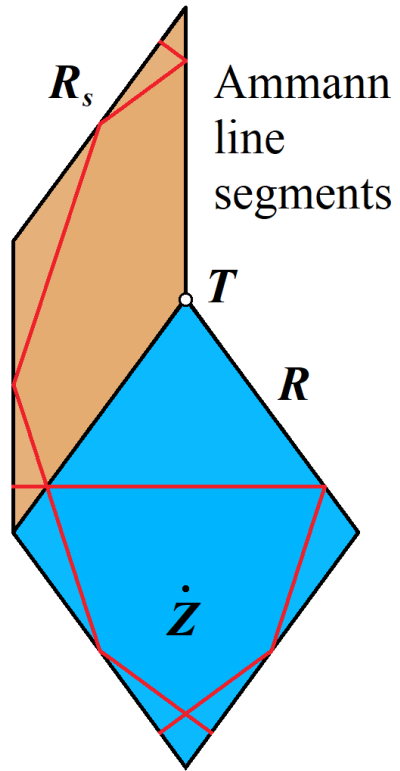
(d)

Rhombus tiling with  $C_3$



(e)

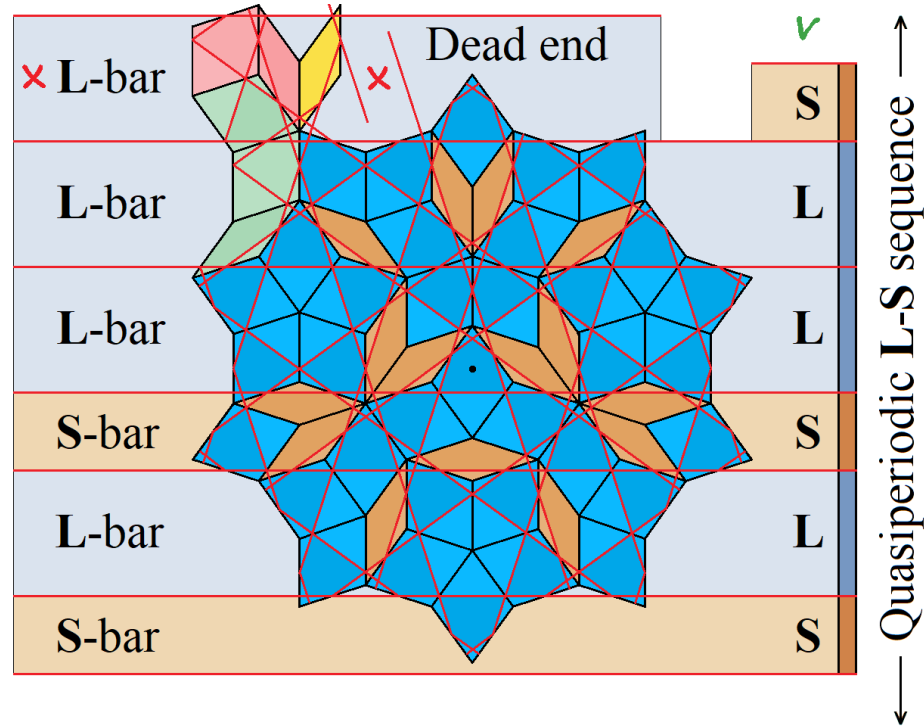
## Matching rules



Proto-rhombs

Figure 2(a)

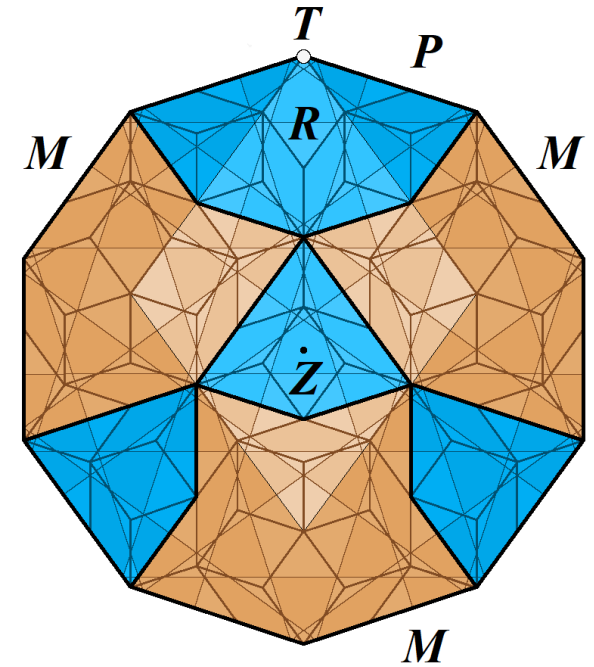
## Rule fulfillment in a cartwheel $C_3$



Continous Ammann lines create a grid  
Above: Incompleteness of the rules

(b)

## Covering rules



Gummelt-Decagon  $P$   
(Rule also incomplete)

(c)

Substitution of the rhombus  $R_q$  and its Ammann lines, shown by the horizontal lines

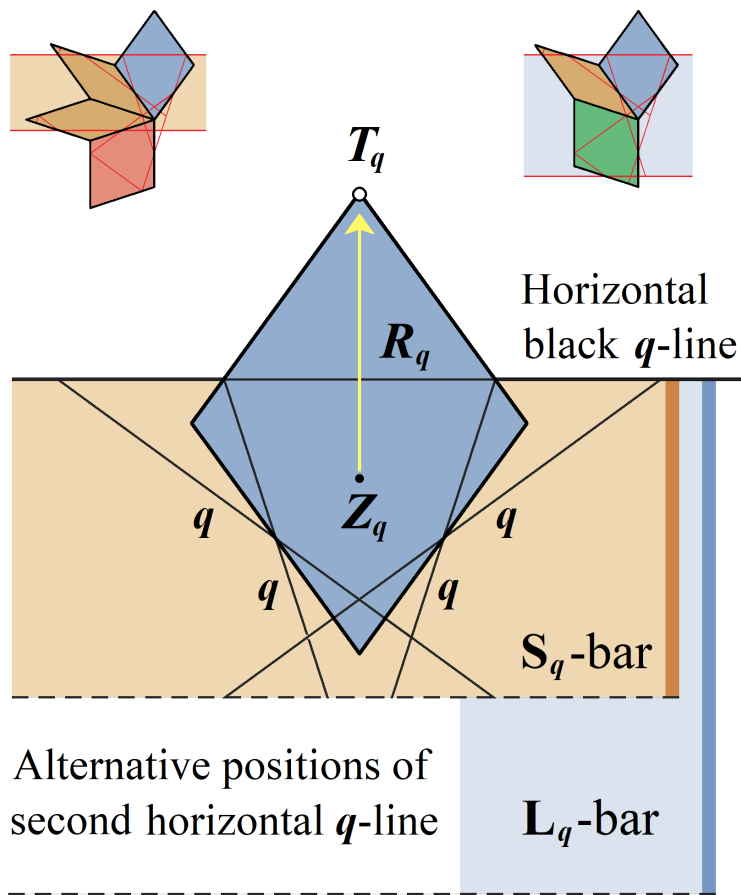
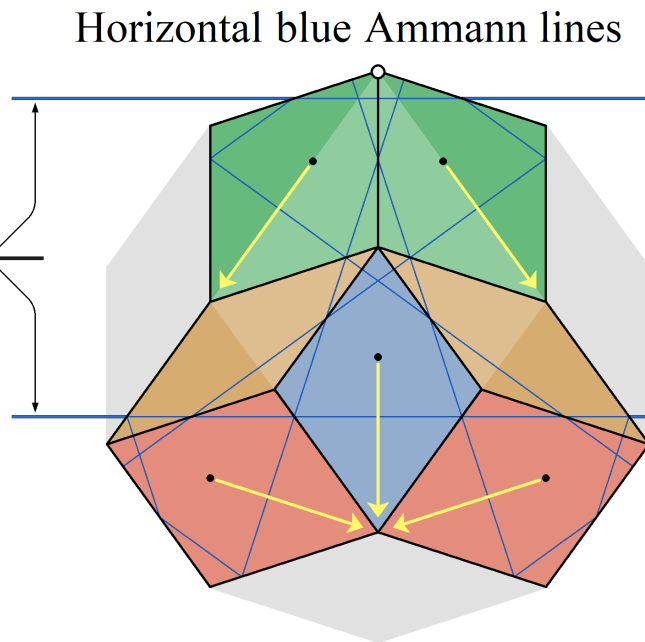


Figure 3(a)

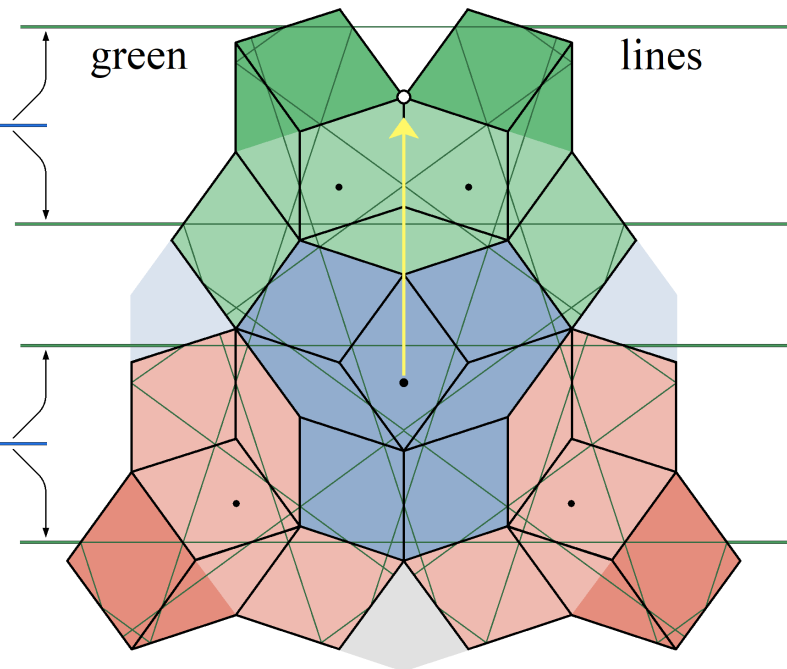
First Substitution



Cartwheel  $C_1$

(b)

Second substitution



Cartwheel  $C_2$

(c)

### Third substitution

### Ammann grid of the quasi-cell $Q$

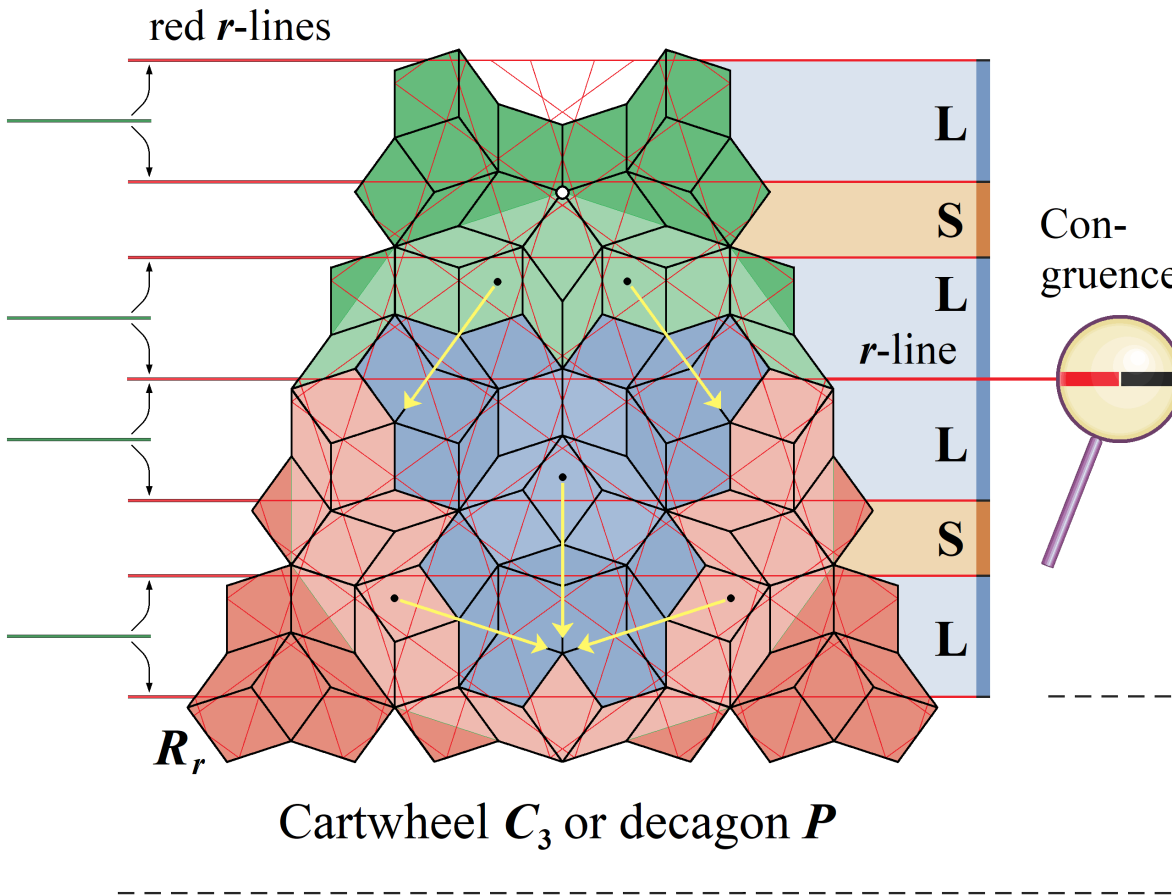
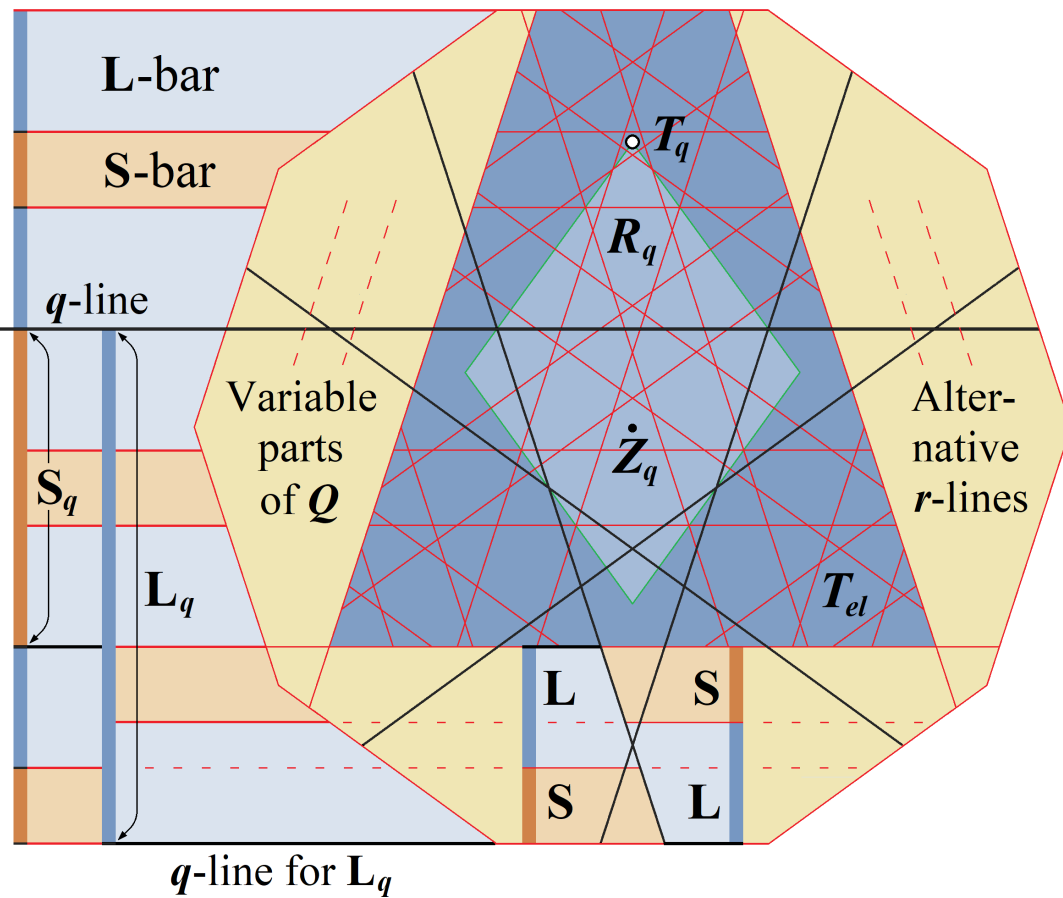


Figure 3(d)



(e)

## Girih pattern with highlighted Girih stencils

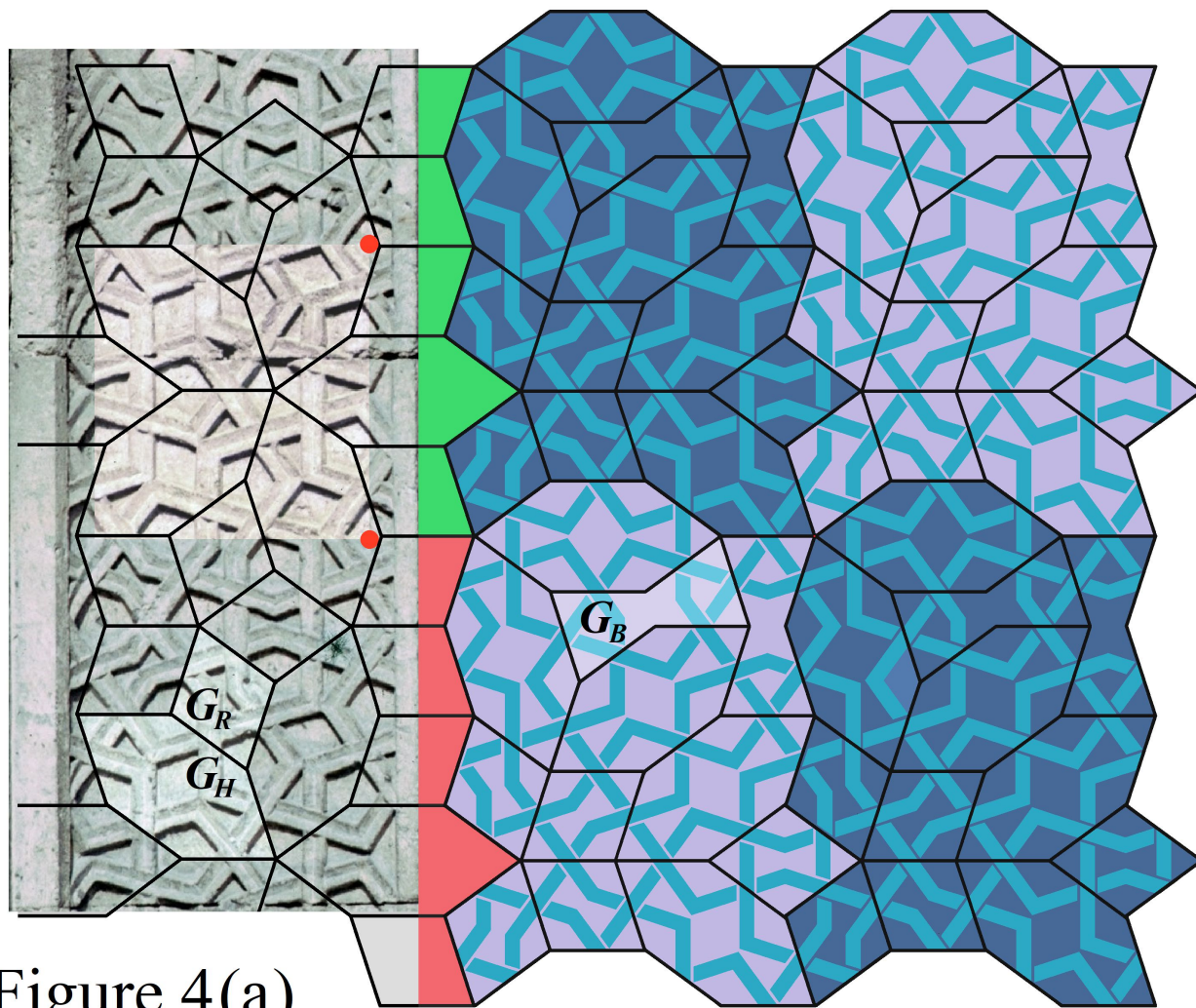
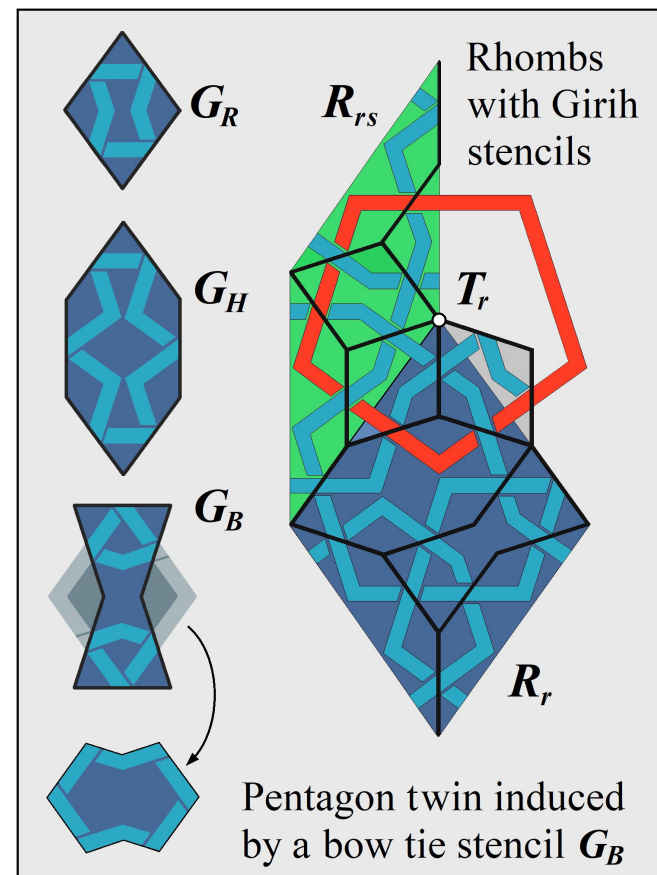


Figure 4(a)

## Incorporation into rhombs



(b)

## Incorporated stencils

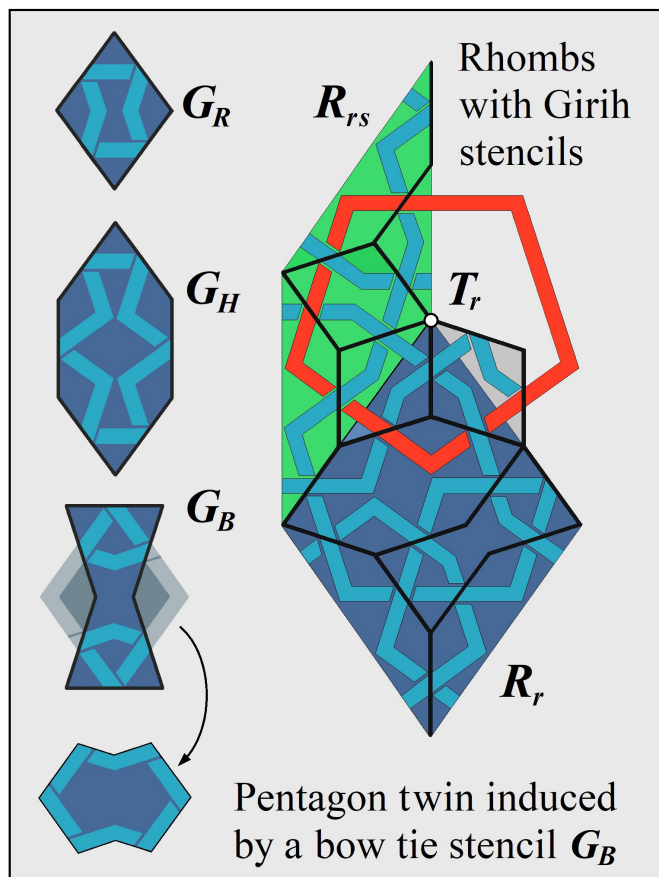
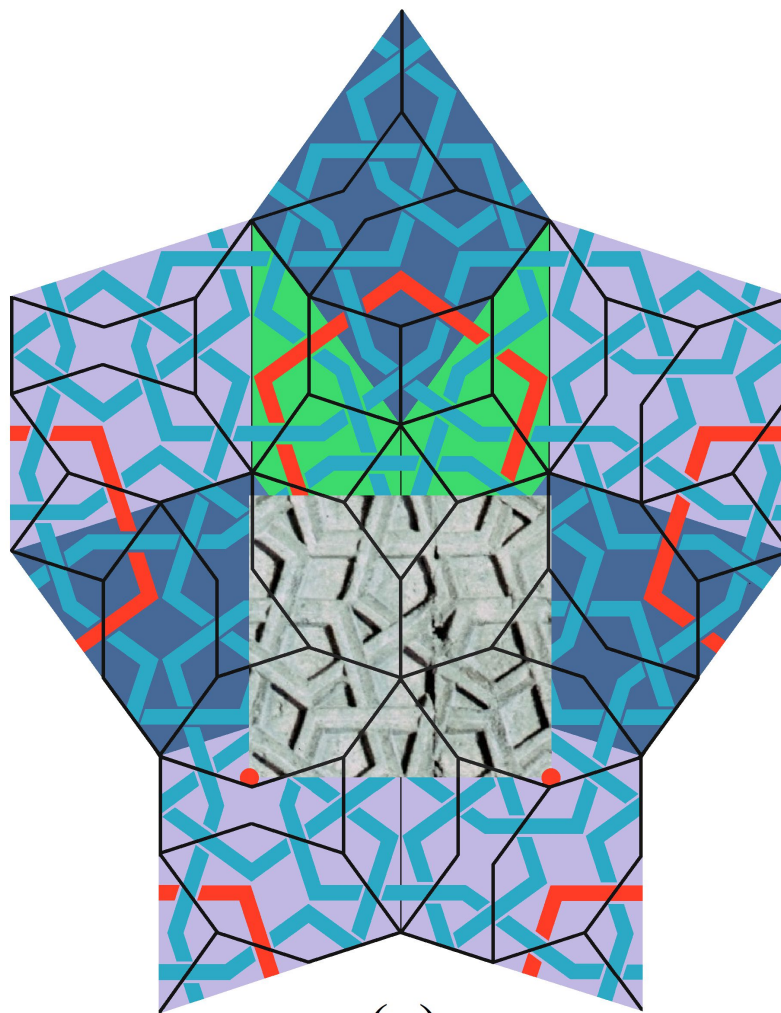


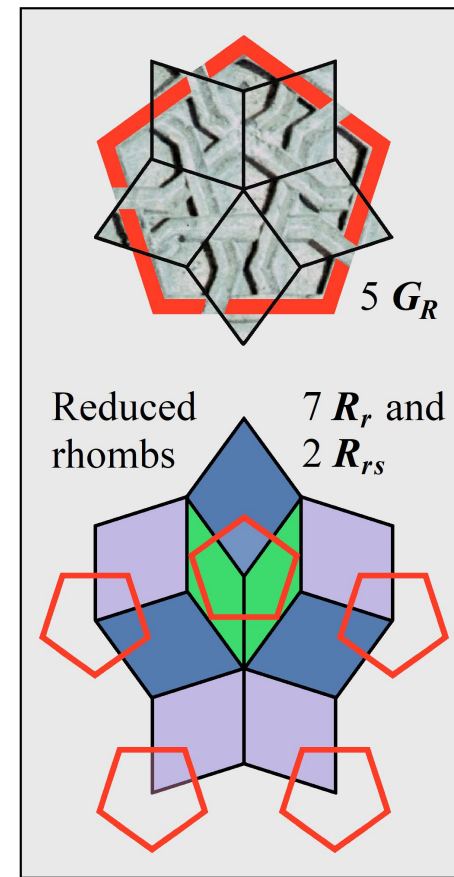
Figure 4(b)

## Quasiperiodic Girih pattern



(c)

## Girih pentagons



(d)



# $Q$ with twin-scales $I$ and $q$ -lines

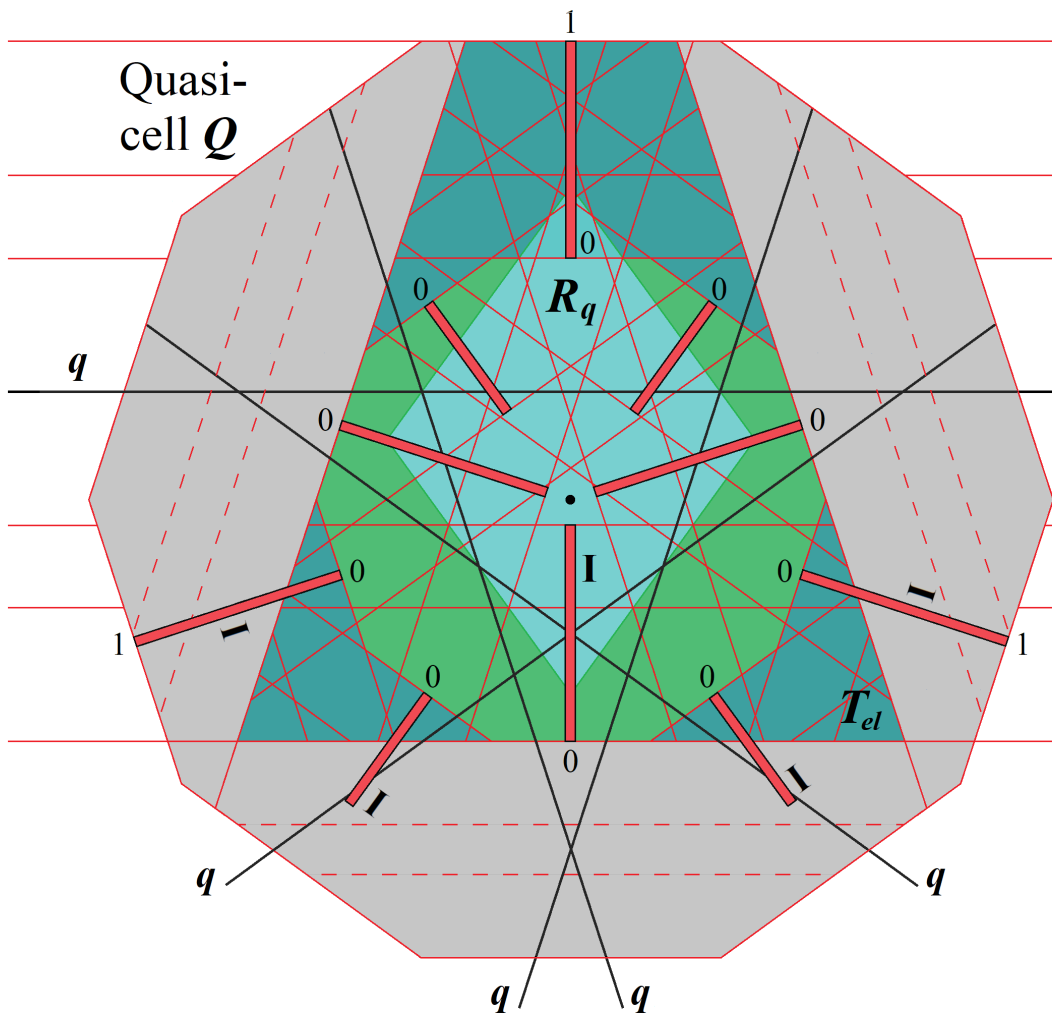
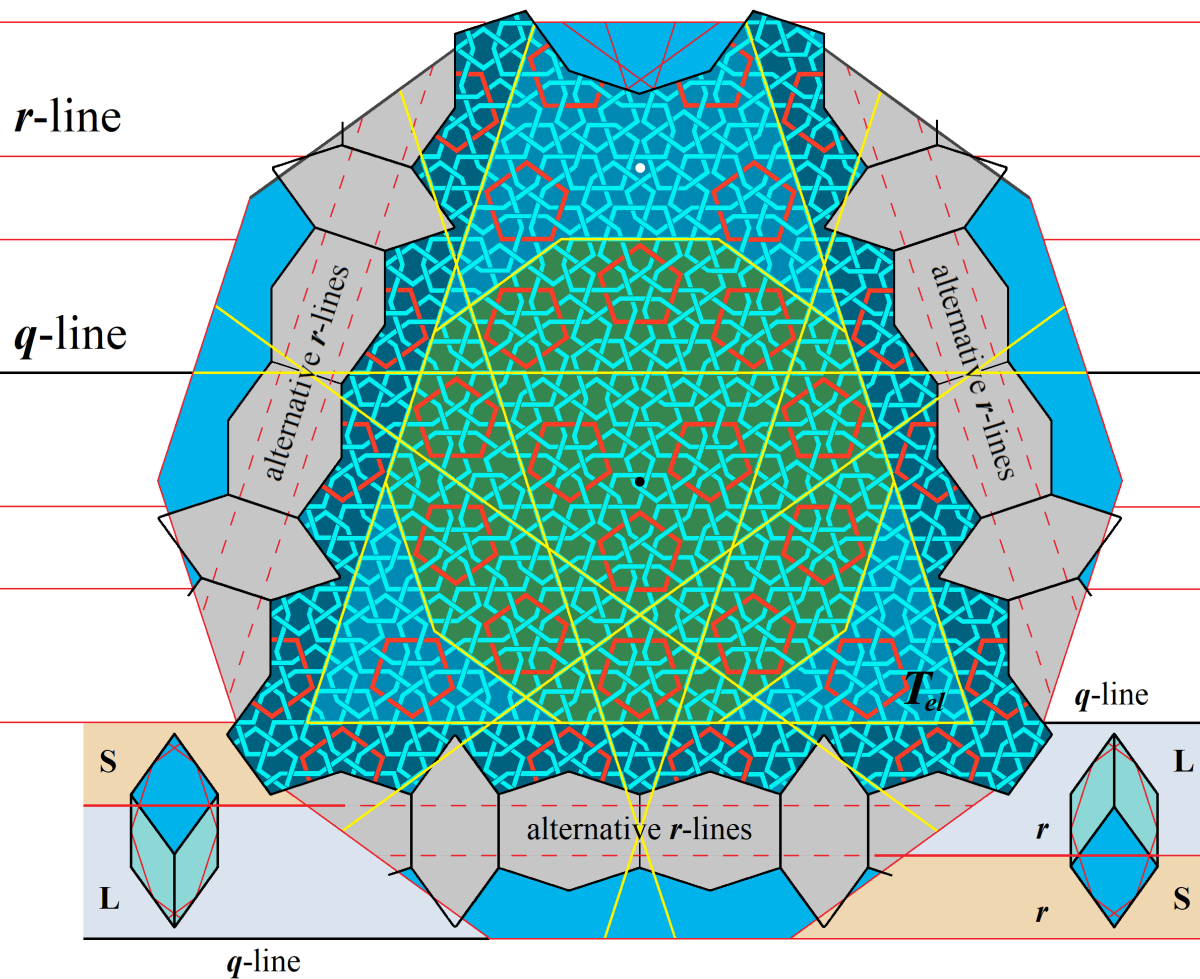
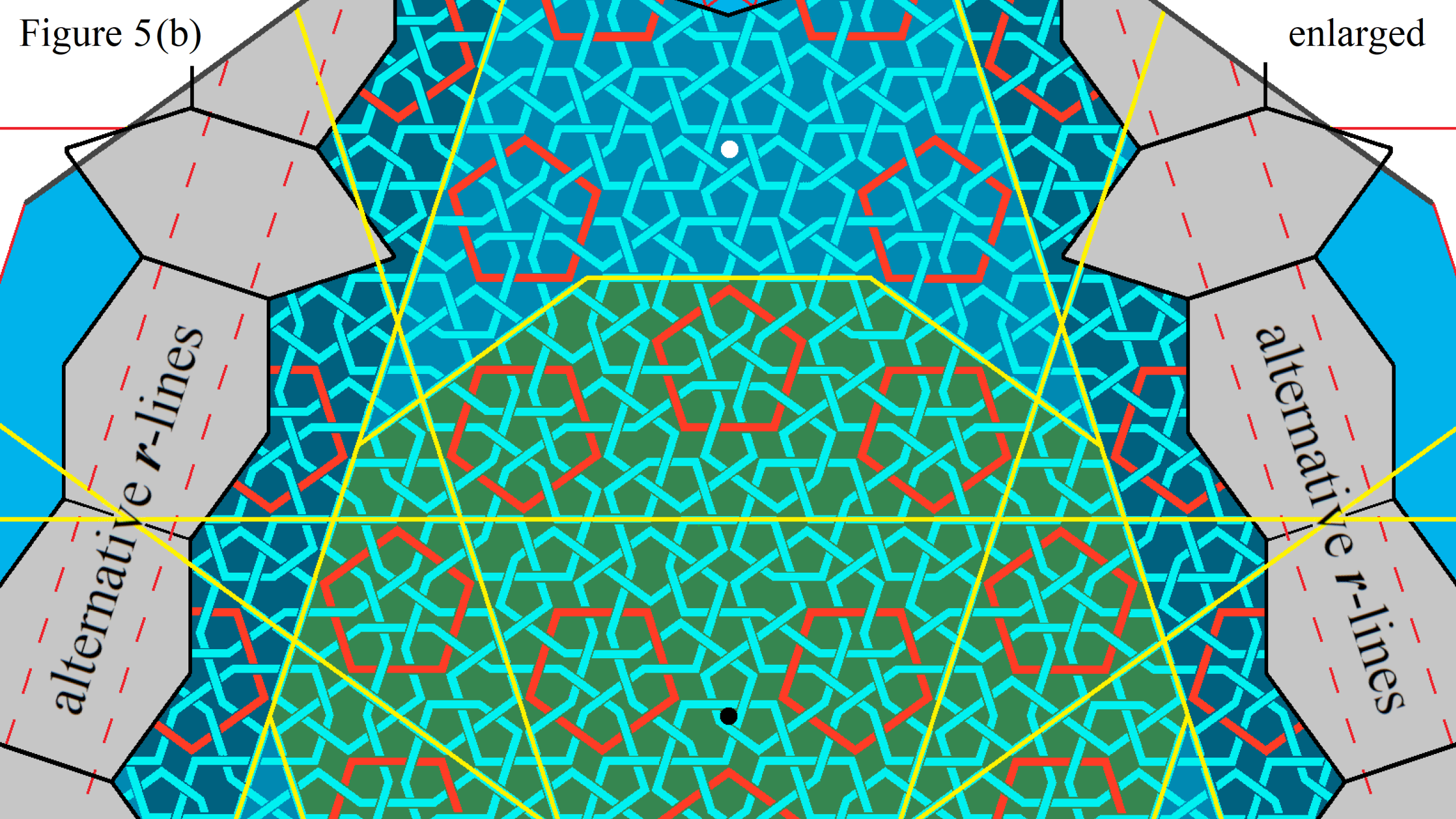


Figure 5 (a)

# $Q$ with Girih pattern and flipping worms



(b)



$Q$  with specified lower flip type

Three specified flips & three Girih knot types

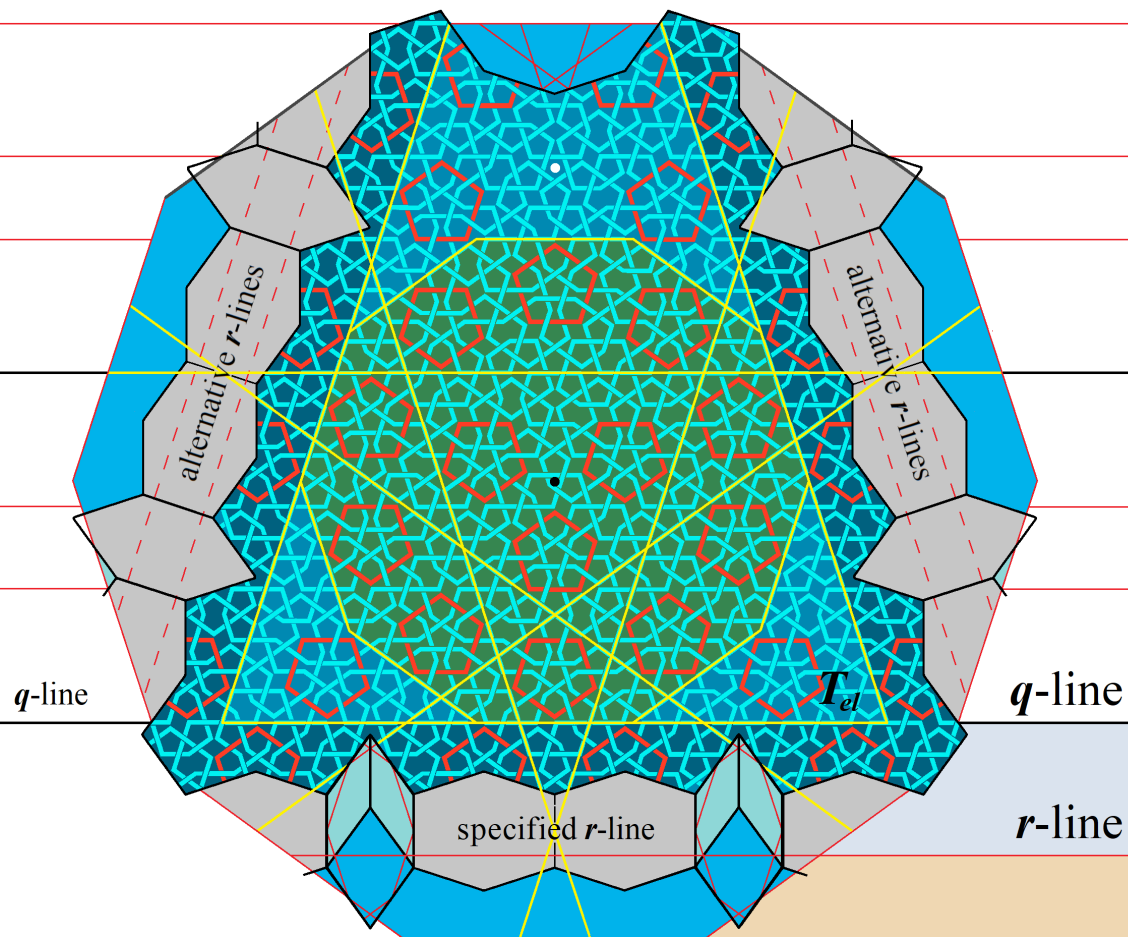
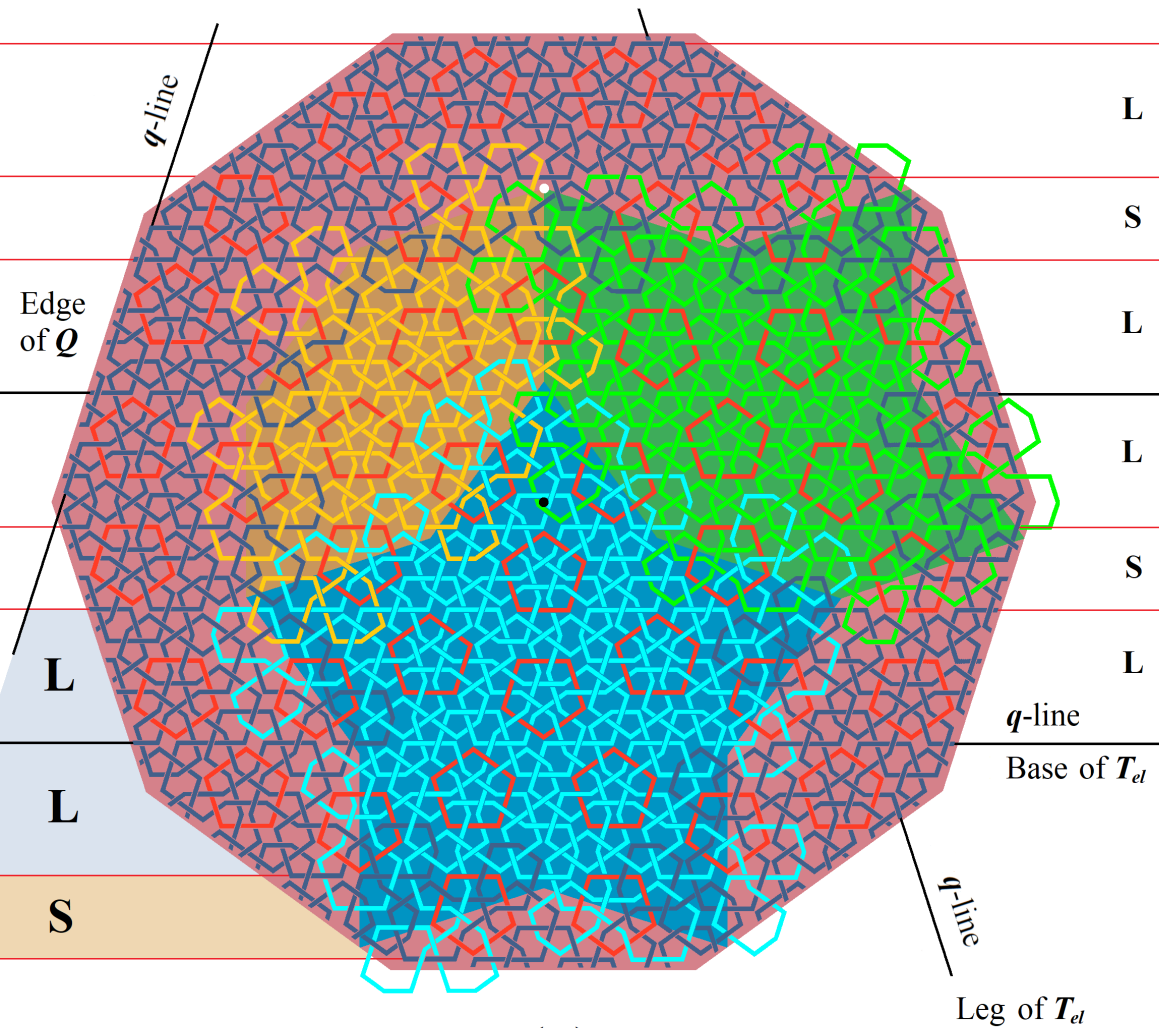
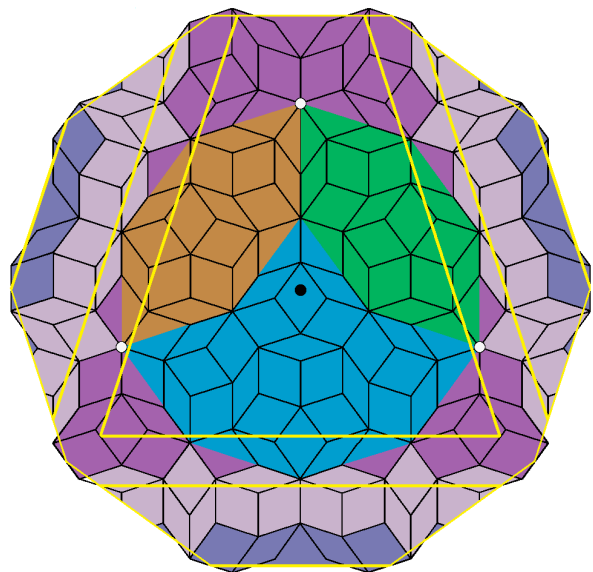


Figure 5 (b\*)

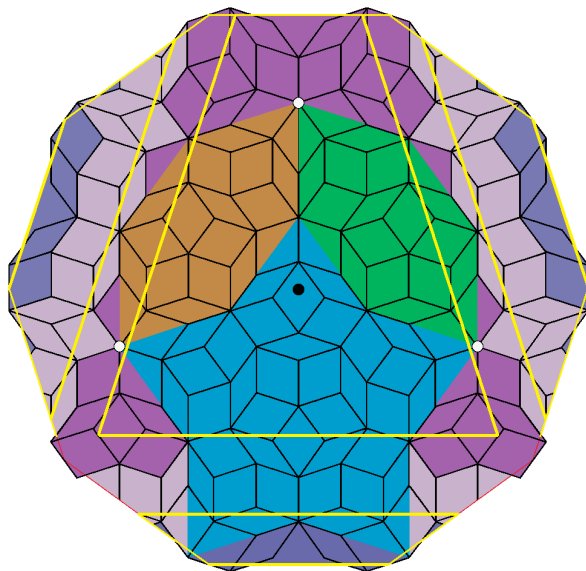


(c)

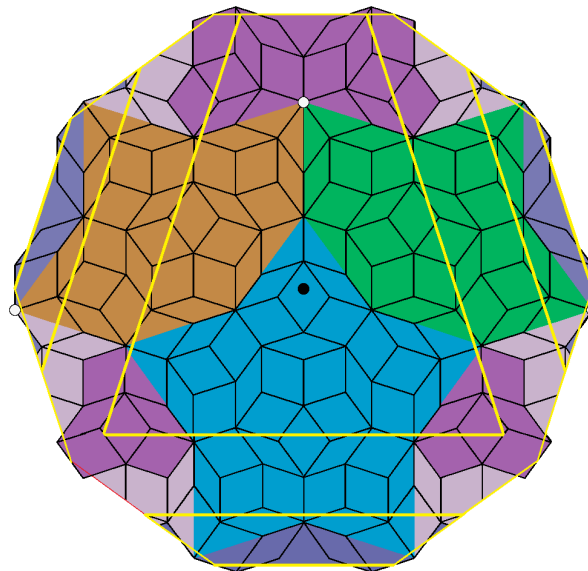
*H-B-H*



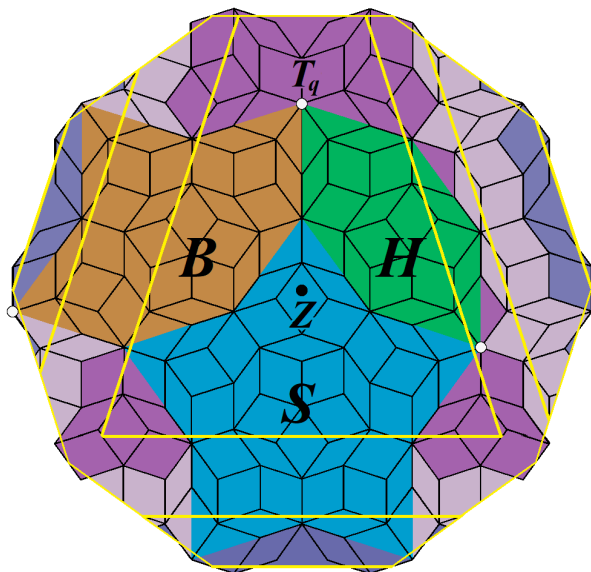
*H-S-H*



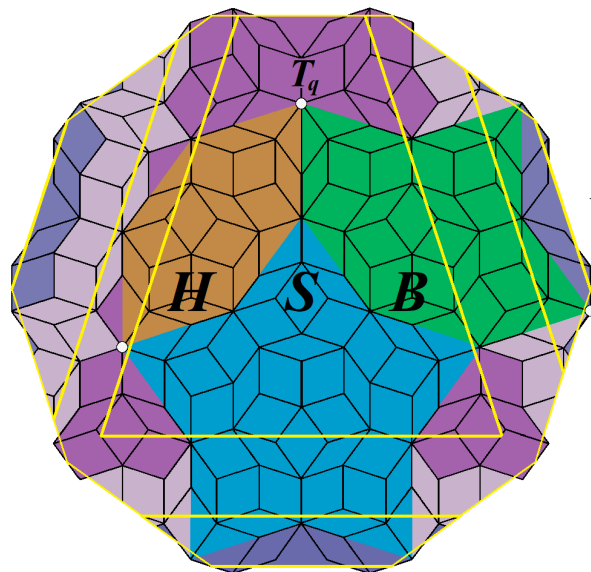
*B-S-B*



*B-S-H*



*H-S-B*



The five status types of the quasi-cell  $Q$

Figure 5(d)

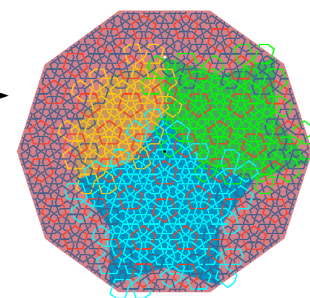


Figure 5(c)

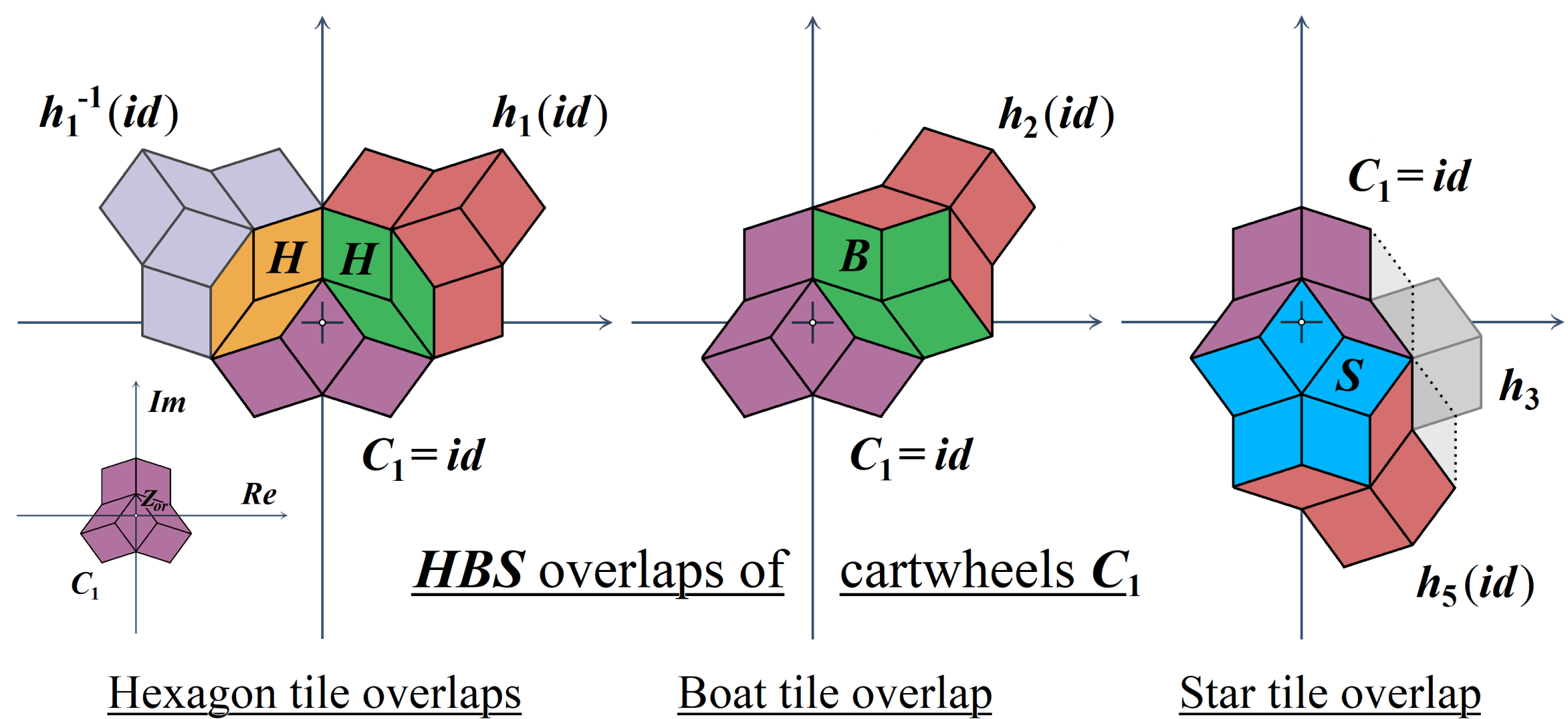


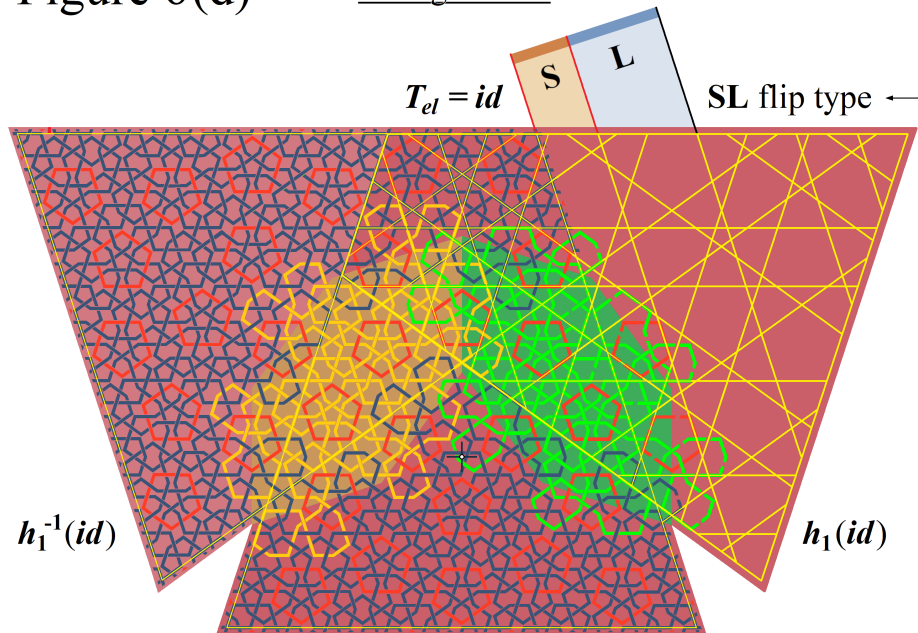
Figure 6(a)

(b)

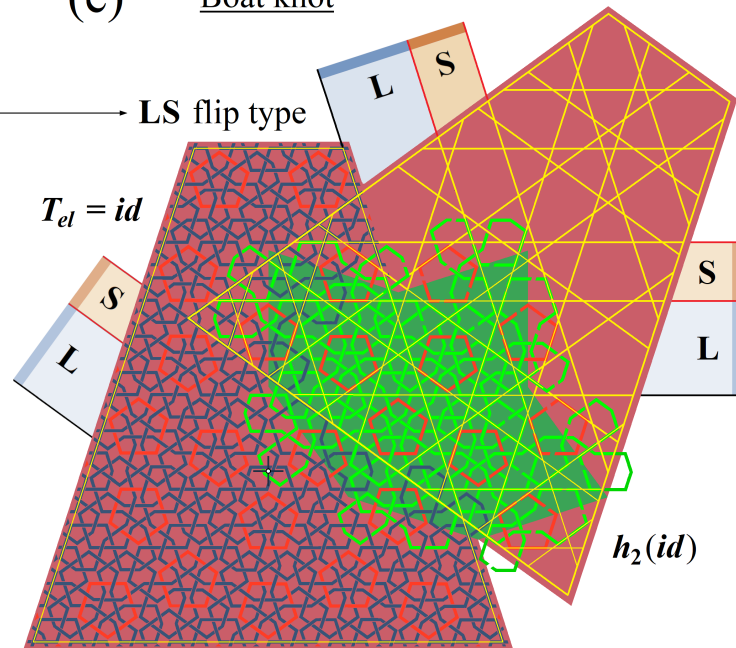
(c)

Figure 6(d)

Hexagon knot

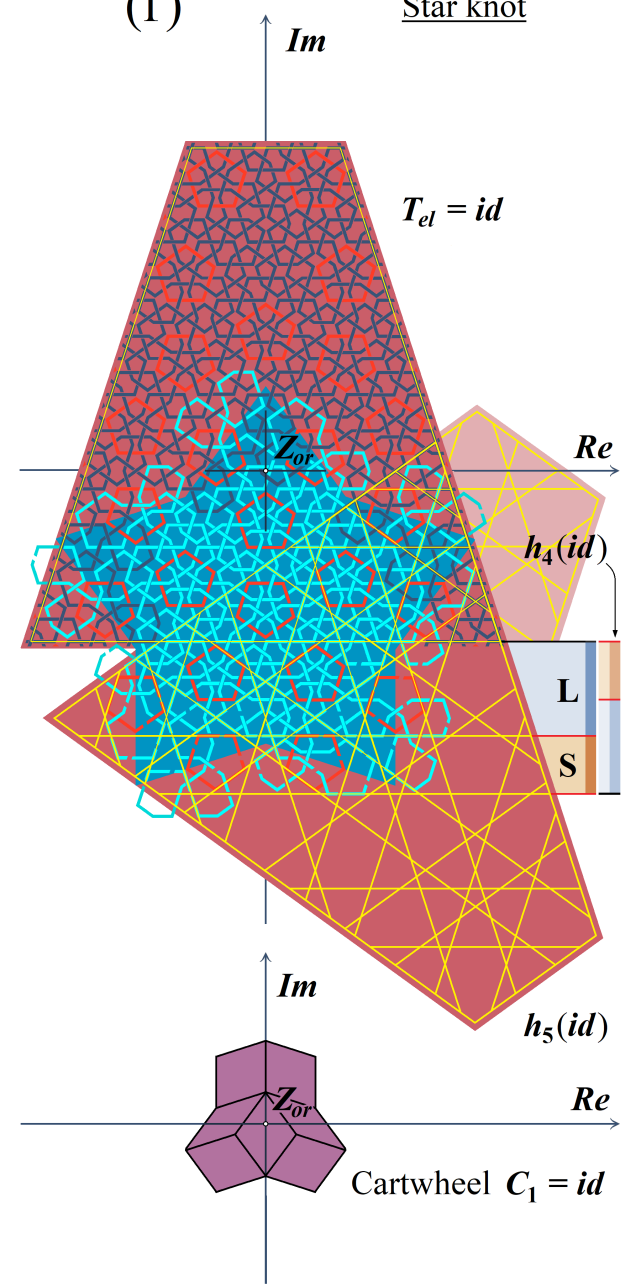


(e) Boat knot

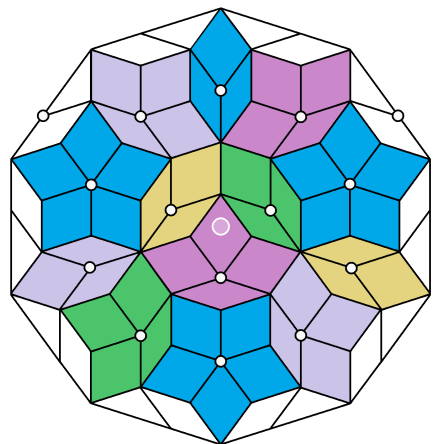


(f)

Star knot

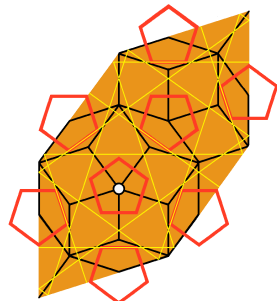


(g) HBS tiles in cartwheel order

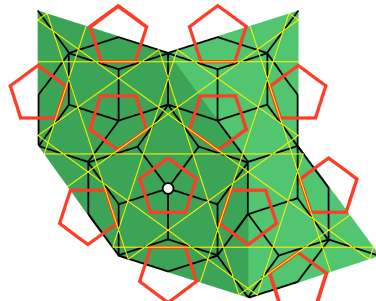


(h) HBS tiles with  $R_r$  rhombus structure, red Girih pentagons and subgrid

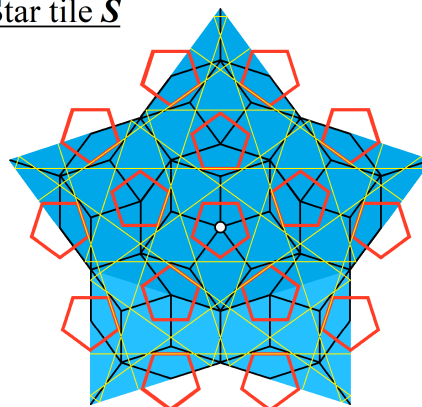
Hexagon tile  $H$



Boat tile  $B$



Star tile  $S$



$Im$

$Re$

Cartwheel  $C_1 = id$

Knot components

Meander

( $b^*$ )

Large loop

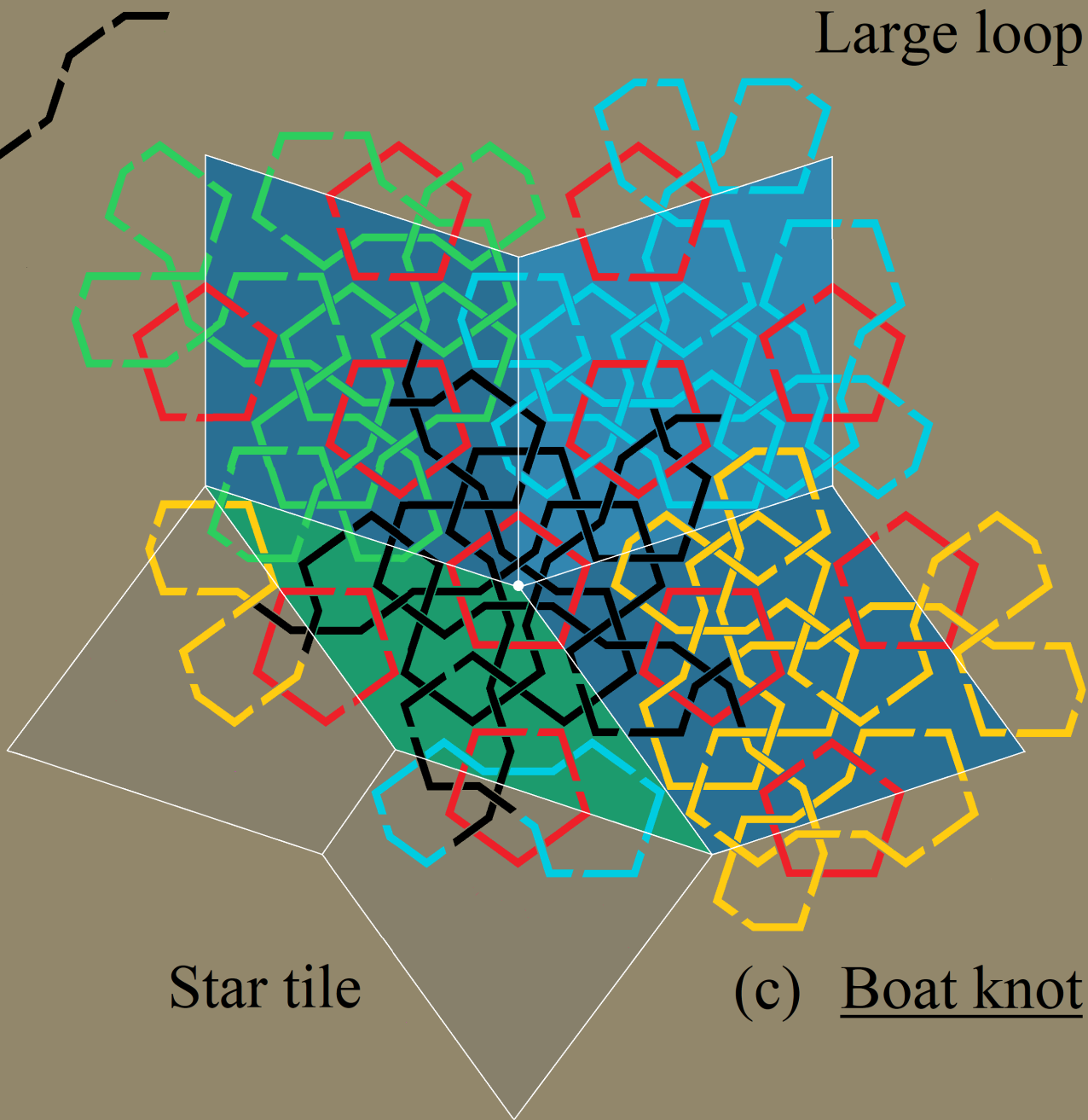
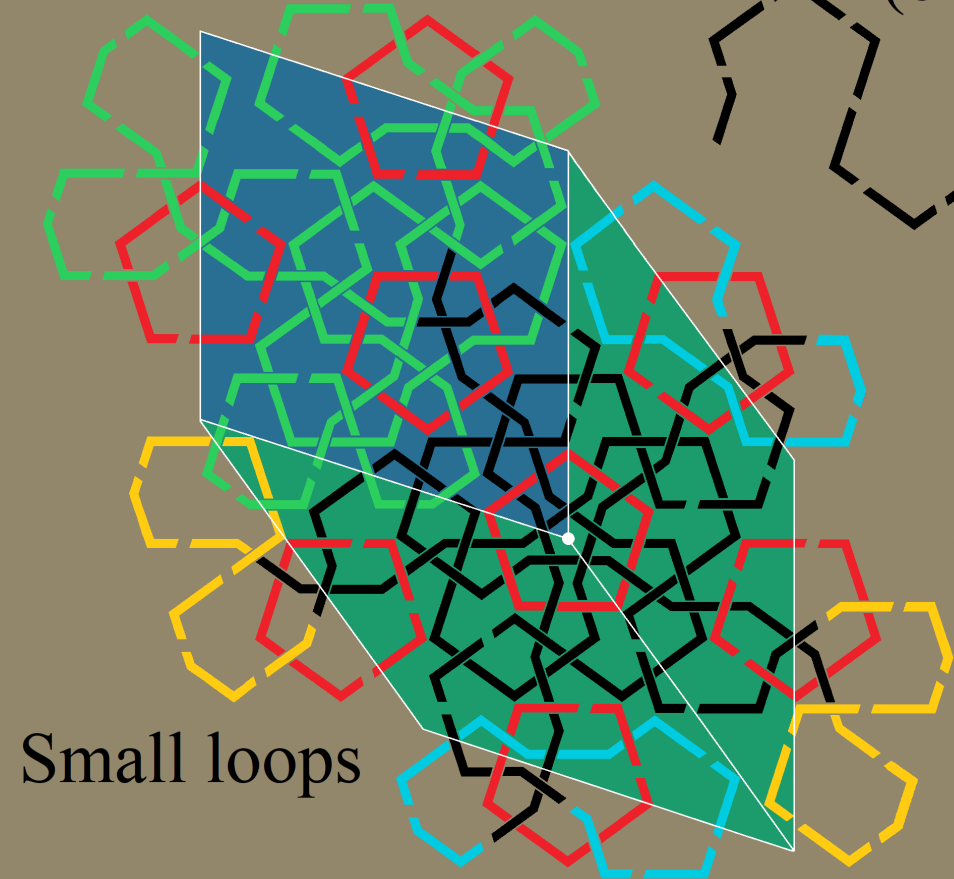
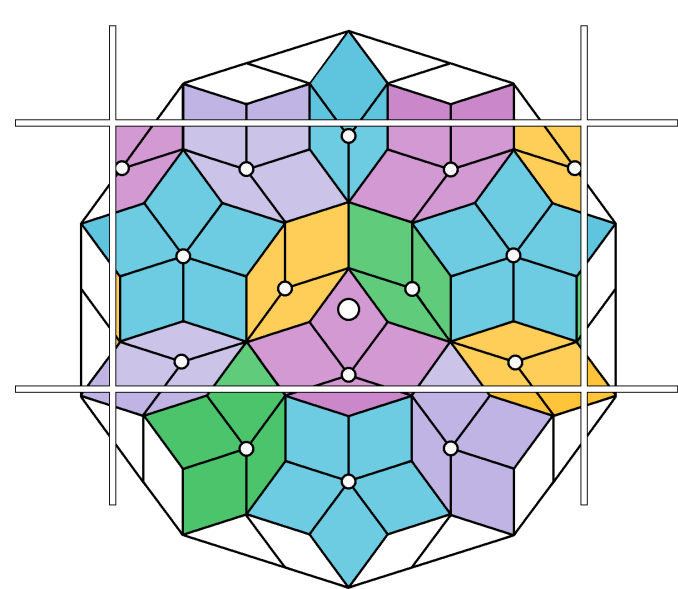


Figure 7(a)

Hexagon knot

Star tile

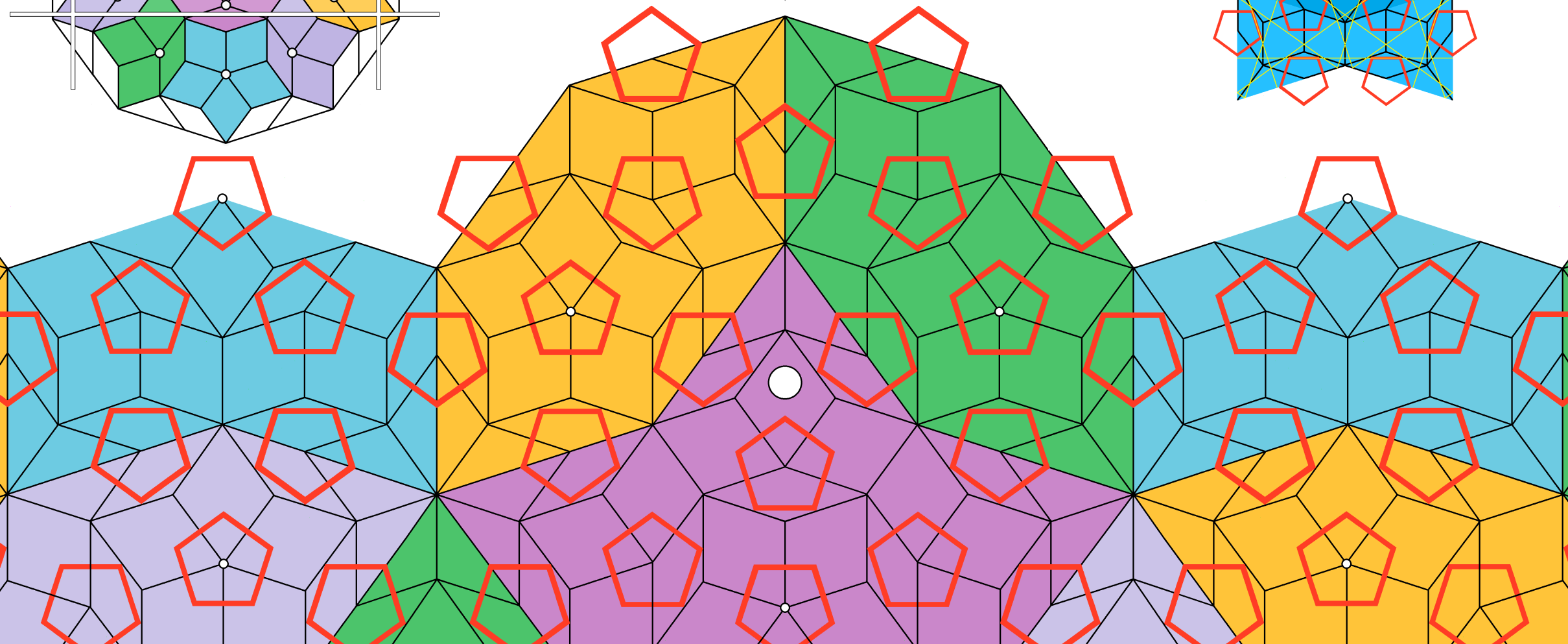
(c) Boat knot



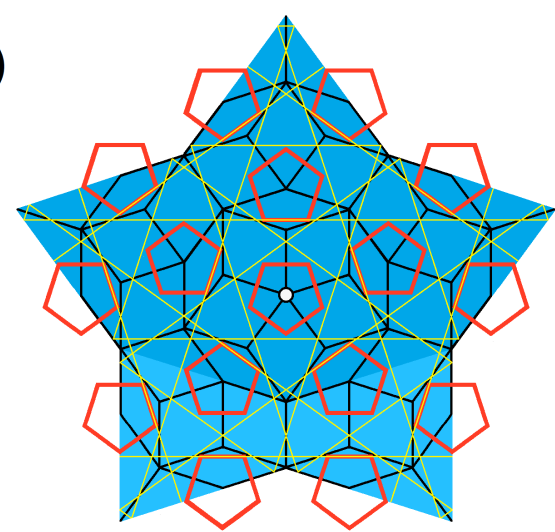
(g)

Figure 6

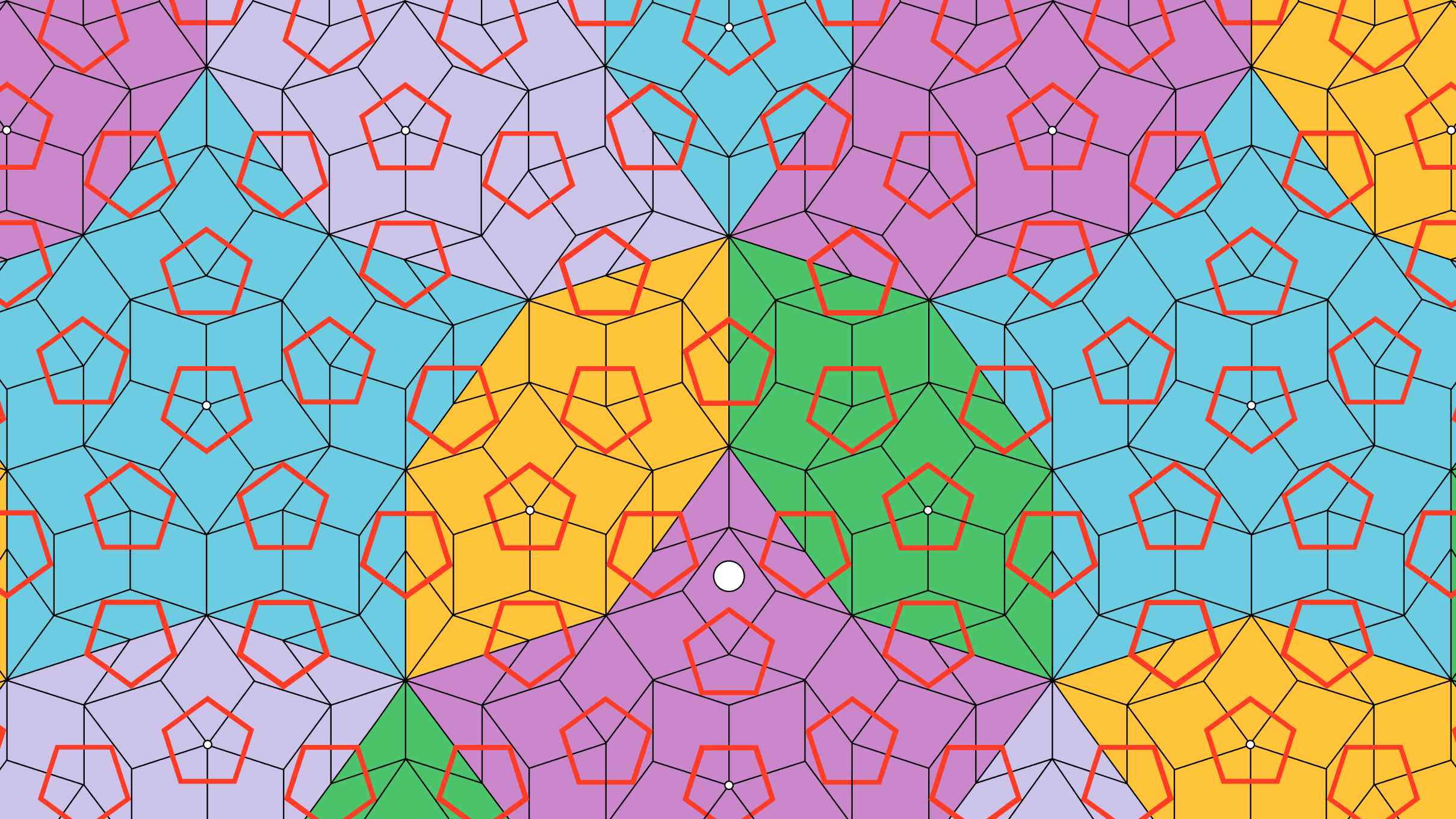
(g)  
enlarged

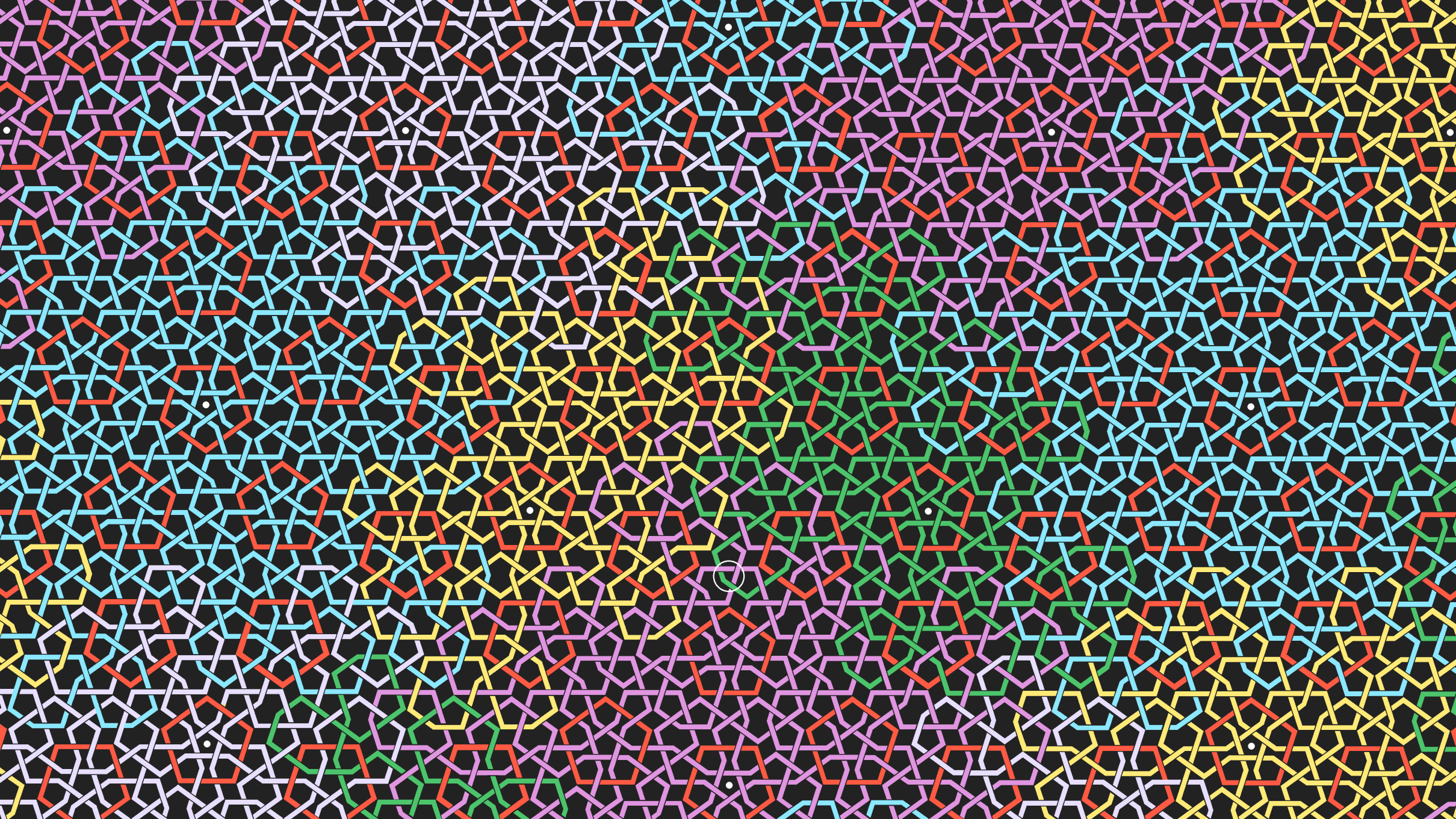


(h)





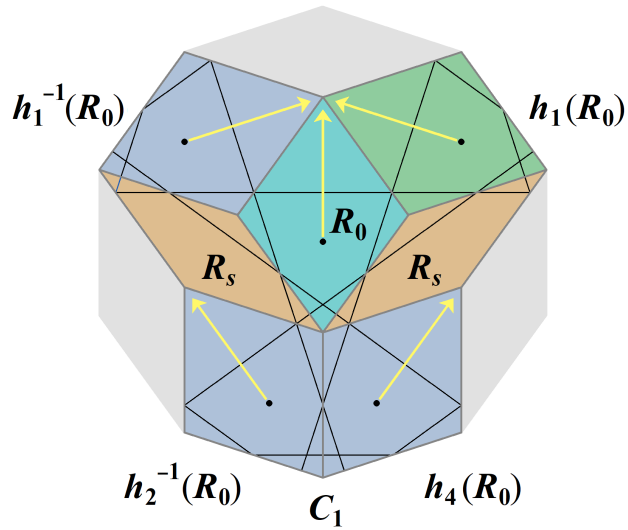




# Admissible and forbidden transformations controlled by the quasiperiodic succession algorithm

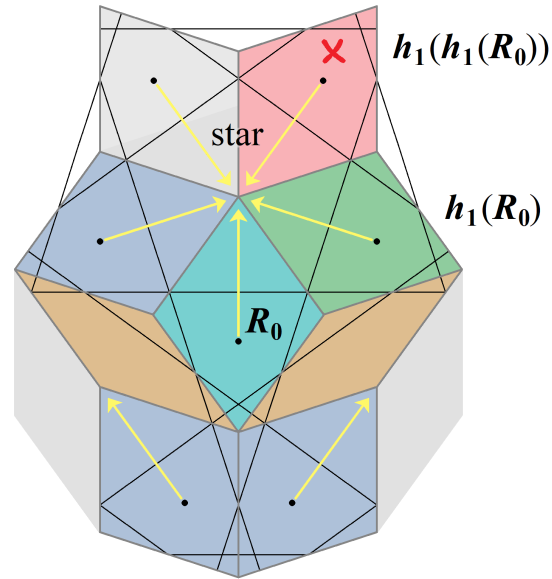
Each rhombus tiling which is generated by the succession algorithm consists on cartwheels nested inside each other with a start rhombus  $R_0$  in the center of  $C_1$ .

Therefore  $R_0$  cannot belong to a rhombus star as shown below.



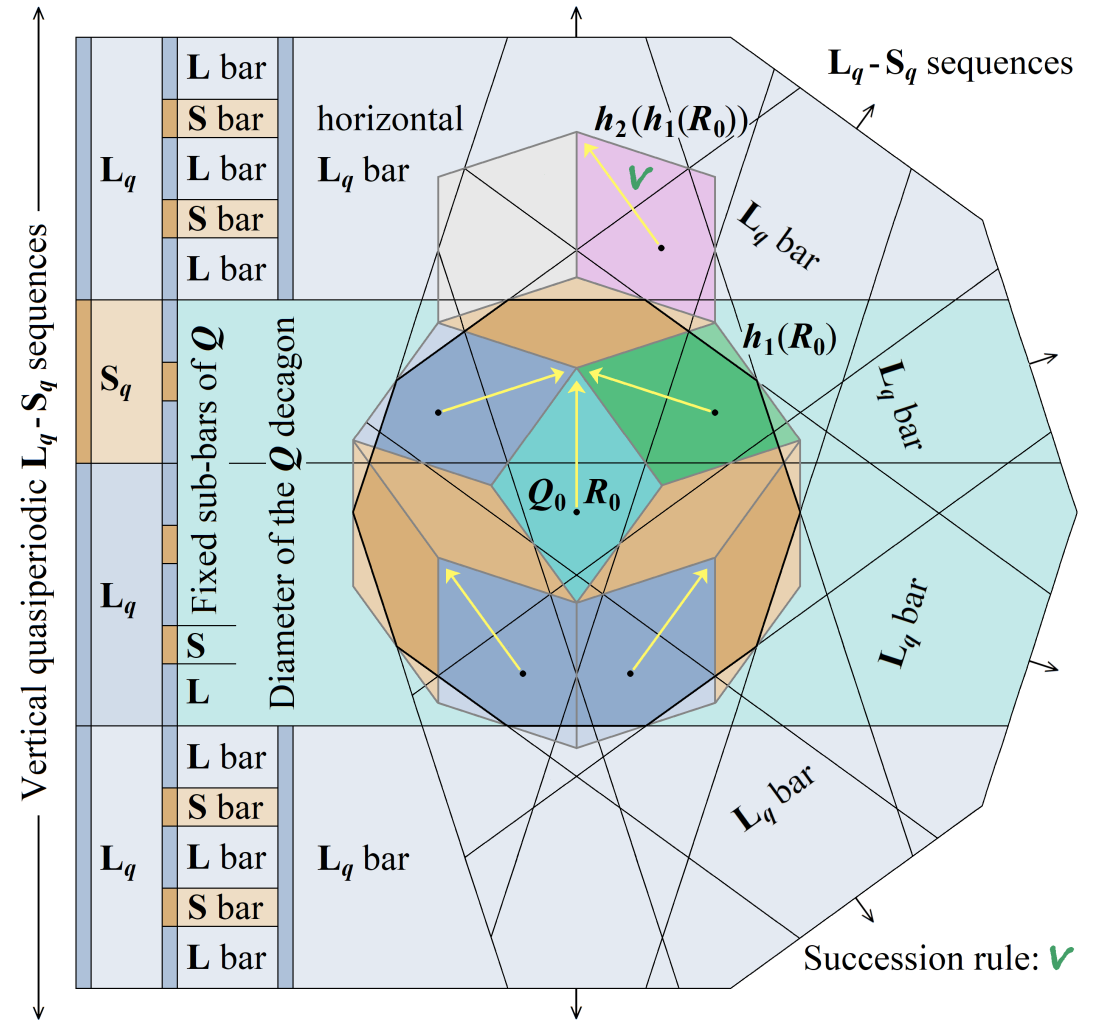
Fulfillment of:  
 Matching rules: ✓  
 Succession rule: ✓

Figure A(a)



Fulfillment of:  
 Matching rules: ✓  
 Succession rule: ✗

(b)



(c)

# Scale value correlation of overlapping quasi-cells $Q$ shown by the transformation $h_1(Q)$

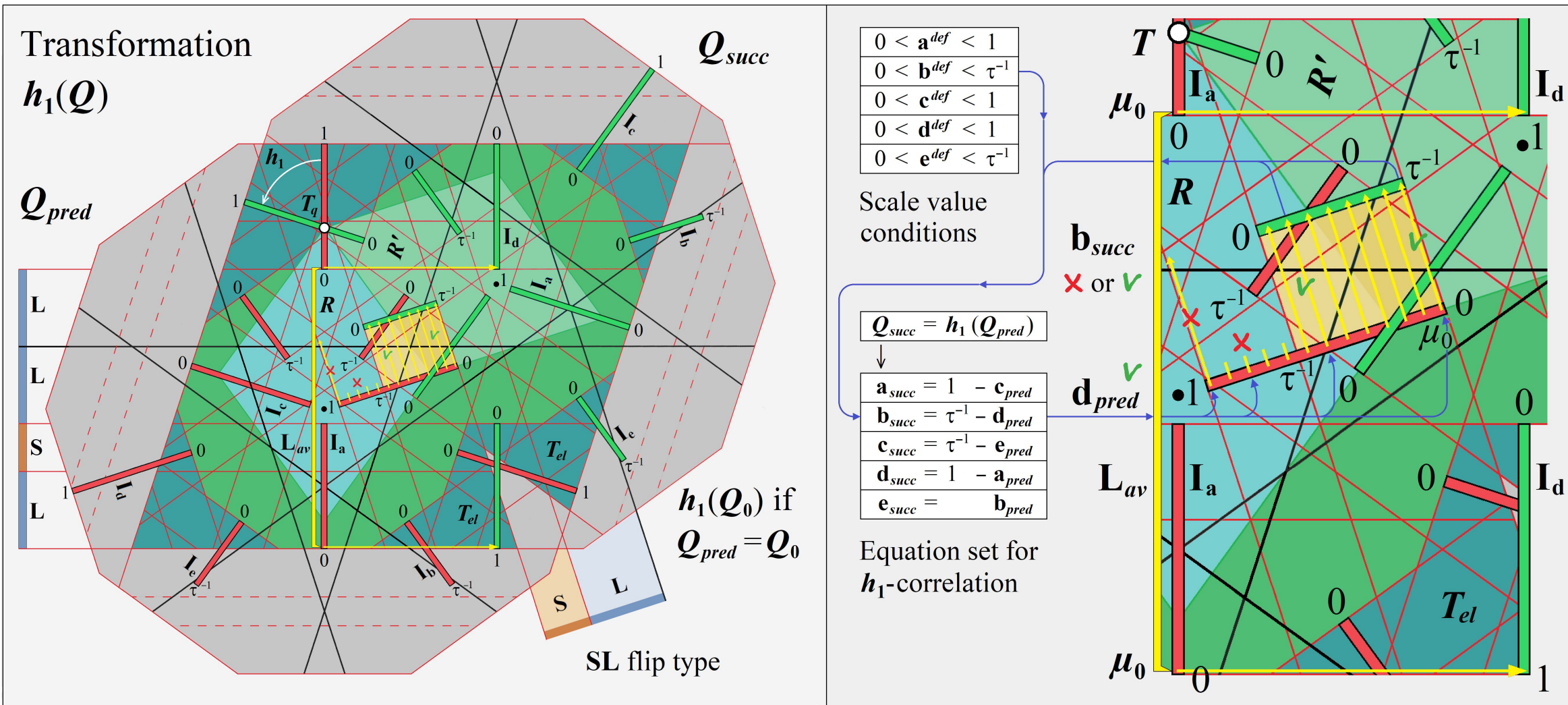


Figure B(a)

(b)