Still life: Five glass surfaces on a tabletop

This image is the result of a collaboration between mathematician Richard Palais and graphic artist Luc Benard. This image was awarded first place in the illustration category of the National Science Foundation / Science Magazine 2006 Visualisation Challenge and was on the cover of the 22 September 2006 issue of Science.

The five mathematical surfaces depicted are (starting from lower-left and moving clockwise): the Klein Bottle, the Symmetric 4-Noid, the Breather Surface, the Boy Surface and the Sievert-Enneper Surface.

The surfaces in the image were created using the 3D-XplorMath program developed by Palais, and assembled and rendered in Bryce by Luc Benard.

Design: Luc Benard, Richard Palais

Wada Basins

This is a rendering that replicates the results of experiments in chaotic scattering. The basic setup is four identical, highly reflective balls sitting in a pyramid formation so that each ball touches every other ball.

If you look into the gaps between three balls, the reflected images you see make up a three dimensional fractal that has what is known as the Wada Property after a Japanese mathematician who studied such spatial objects in 1917.

The Wada Property refers to a discrete dynamical system having three basins of attractions that are so intertwined that every point on one basin boundary is also on the boundary of all other basins. The image was assembled and rendered in Bryce.

Design: Luc Benard

A triply-periodic level surface

The green-tinted surface pictured here has three orthogonal translational symmetries. It is the level-surface given by the trigonometric equation:

$$4*(\cos x*\cos y+\cos y*\cos z+\cos z*\cos x) - 3*\cos x*\cos y*\cos z = -2.4$$

A unit cell may be viewed as a central chamber with tubes to the corners and faces of the cube. This surface closely approximates the minimal P-Surface discovered in 1880 by Karl Hermann Amandus Schwartz (who was a Professor at Halle, Göttingen and Berlin).

Recently it has been investigated by material scientists, who use it and related surfaces to model so-called block-copolymers. The original model was taken from David A. Hoffman and was textured before being placed in a scene that we feel enhances the beauty of the object; it was rendered in Bryce.

Design: Luc Benard

Equations Studies

These fractal images were produced by using various equations as input in Stephen Ferguson's Flarium24 Windows program, using 40 iterations and the filter

```
rr+=atan(fabs(dzy/dzx))*atan(fabs(dzx/dzy))*2
```

They were then assembled in Photoshop using fonts and coloration to mimic the old technical drawings of Leonardo DaVinci. While these images are highly mathematical in origin, many peoples find them aesthetically appealing completely independent of their source.

Design: Luc Benard

Lyapunov Play

Dynamical systems can be used to study the evolution of animal populations - the change over time of food, fertility, size, etc. - with dynamics requiring the ability of reproduction to alternate quasi-periodically between two values. Such systems can show both a stable cycle and chaotic evolution depending on the fertility ability. Stability or chaos can be analysed by computing the so-called Lyapunov exponent. (Lyapunov was a 19th century Russian mathematician.)

Markus-Lyapunov images are colour mappings of the Lyapunov exponent versus fertility, along horizontal and vertical axes. Only the stability domain is plotted; here, chaos is rendered in dark blue. As the exponent goes from 0 to minus infinity, shades range from light to dark. At zero, the chaos threshold, the colour suddenly jumps from dark blue to a lighter shade. There is clearly much that is arbitrary in this colour mapping, and this gives an opportunity for choices based on aesthetic considerations. The picture consists of seven original Markus-Lyapunov pictures which were rebuilt and superimposed.

Design: Luc Benard