

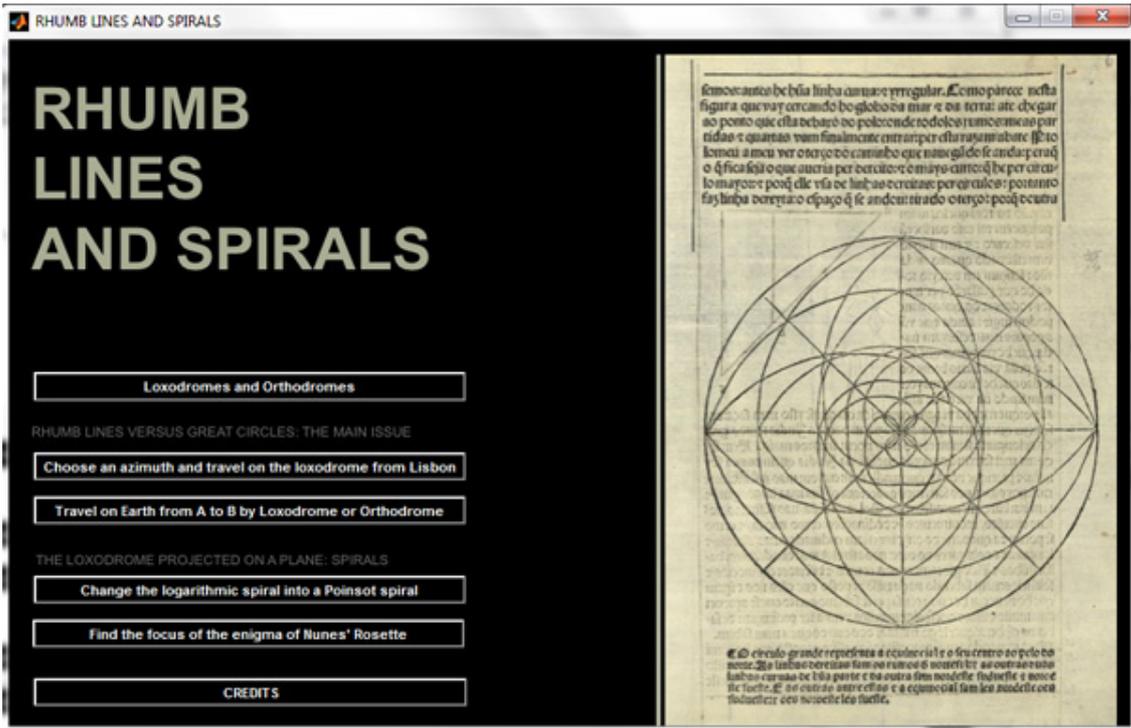
# RHUMB LINES AND SPIRALS

This interactive module illustrates, in a historical context, the rhumb lines on the Earth and the spirals that correspond to a remarkable family of their projections in the plane. It is aimed for Museums and Science Centers, but it may be freely installed in individual computers, under certain specifications, and used for popularization and didactical purposes in public sessions.

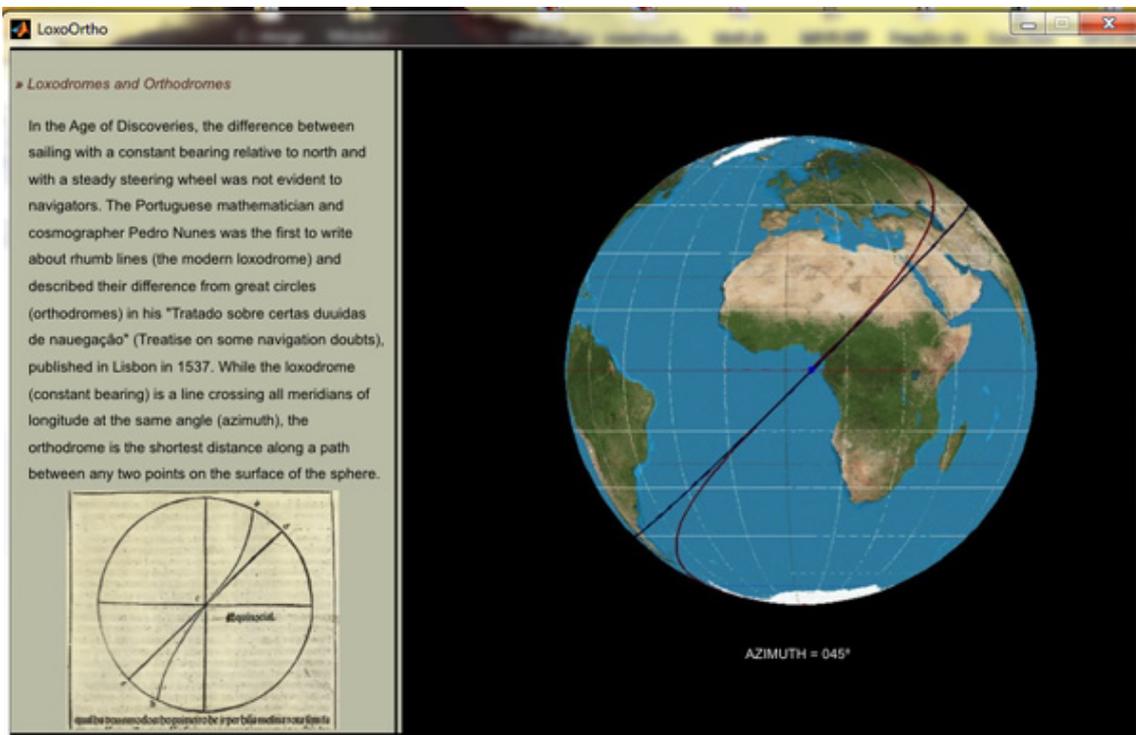
It was created by the CENTRO DE MATEMÁTICA E APLICAÇÕES FUNDAMENTAIS of the University of Lisbon, Portugal, and is submitted to the MPE2013 competition with the open source license Creative Commons BY-NC-ND. The team was composed by Bruno Almeida, Carlos Albuquerque, Rui Lourenço, Sara Rodrigues and José Francisco Rodrigues, who directed the development of the original concept.

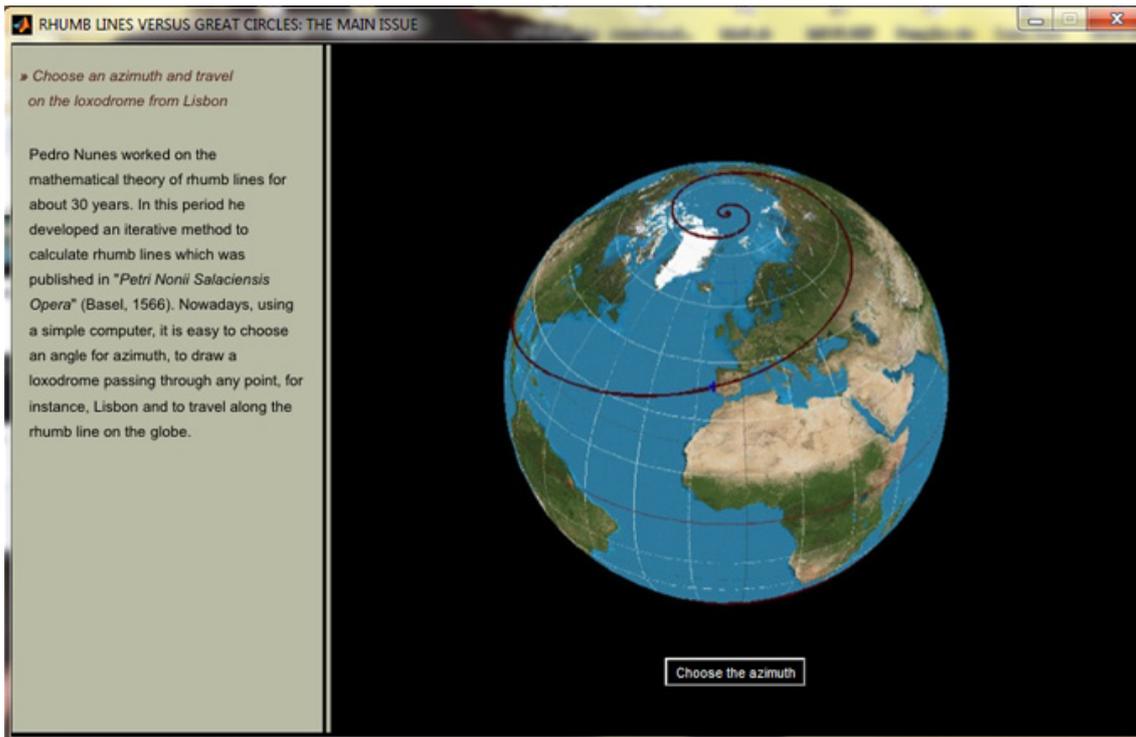
## **DESCRIPTION**

The HOME contains the center of interaction with all the contents, which consist of two chapters, one concerning loxodromes on the globe, the other one spirals. Each chapter has a descriptive introduction leading to two applets. Each page is easily open from the HOME, but it may take a few seconds to run the program. By using the closing button  the user returns to the HOME that should be kept as the open window for navigation.

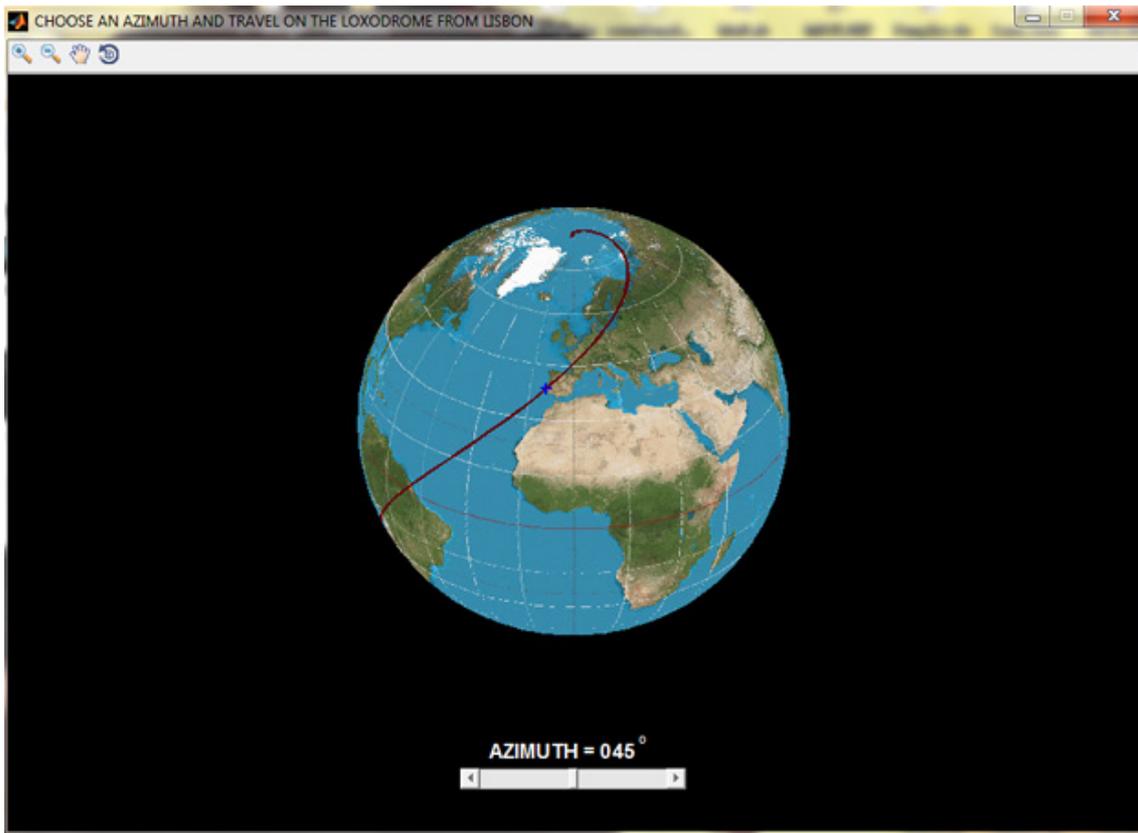


The module starts with the introduction, by opening the window LOXODROMES AND ORTHODROMES the reader may compare a modern illustration of their difference at the azimuth  $45^\circ$  with the first historical representation of 1537, by Pedro Nunes.

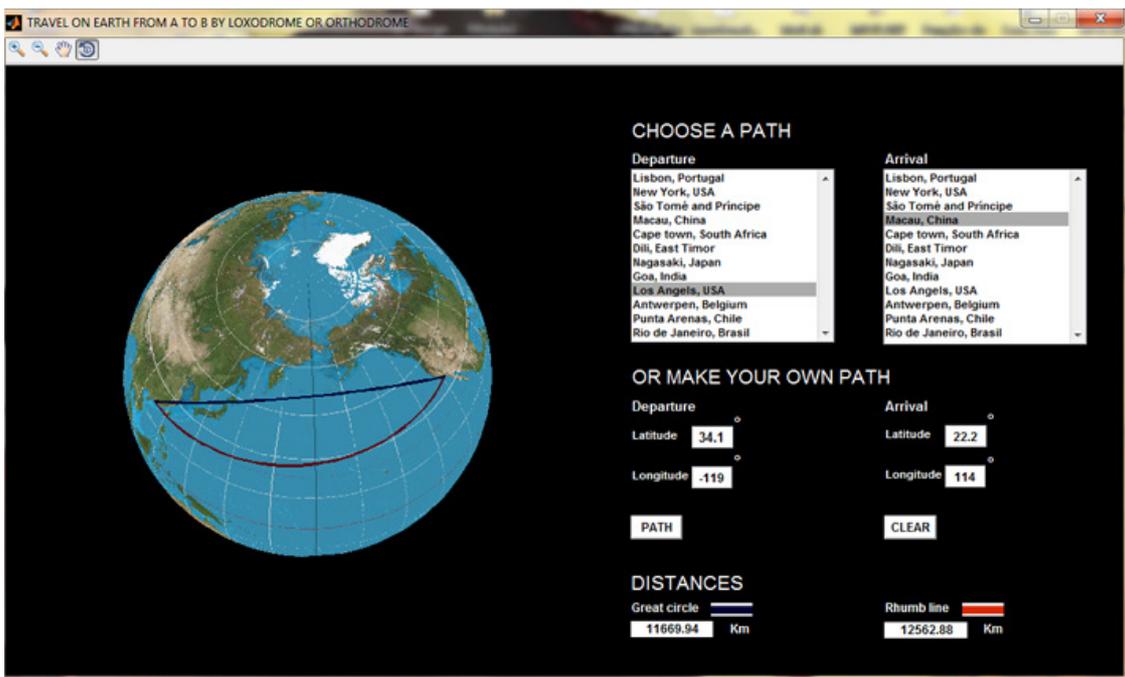




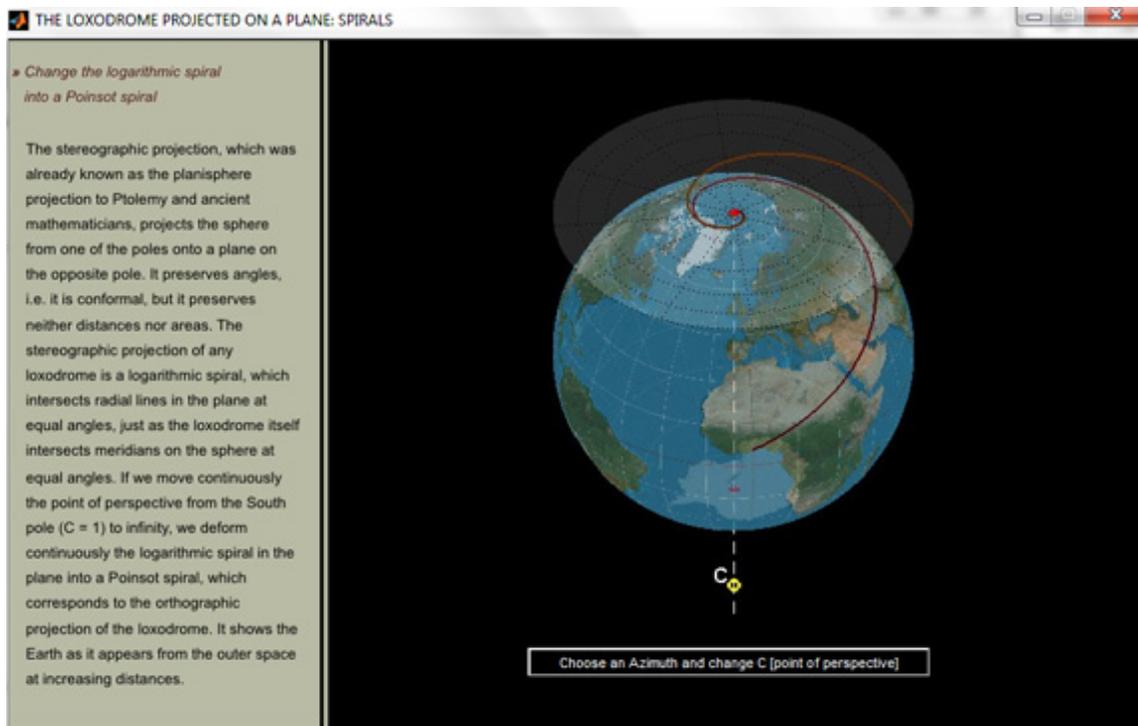
The first chapter on RUMB LINES VERSUS GREAT CIRCLES introduces the main issue understood by the Portuguese mathematician in the 16<sup>th</sup> century in a window that leads to the first applet, by clicking at **Choose the azimuth** . It shows how the form the LOXODROME takes after choosing the angle with the North, the azimuth. With the window CHOOSE AN AZIMUTH AND TRAVEL ON THE LOXODROME FROM LISBON, the user may change the slider  by changing the azimuth which starts at 45° and ranges from 0° up to 90°.



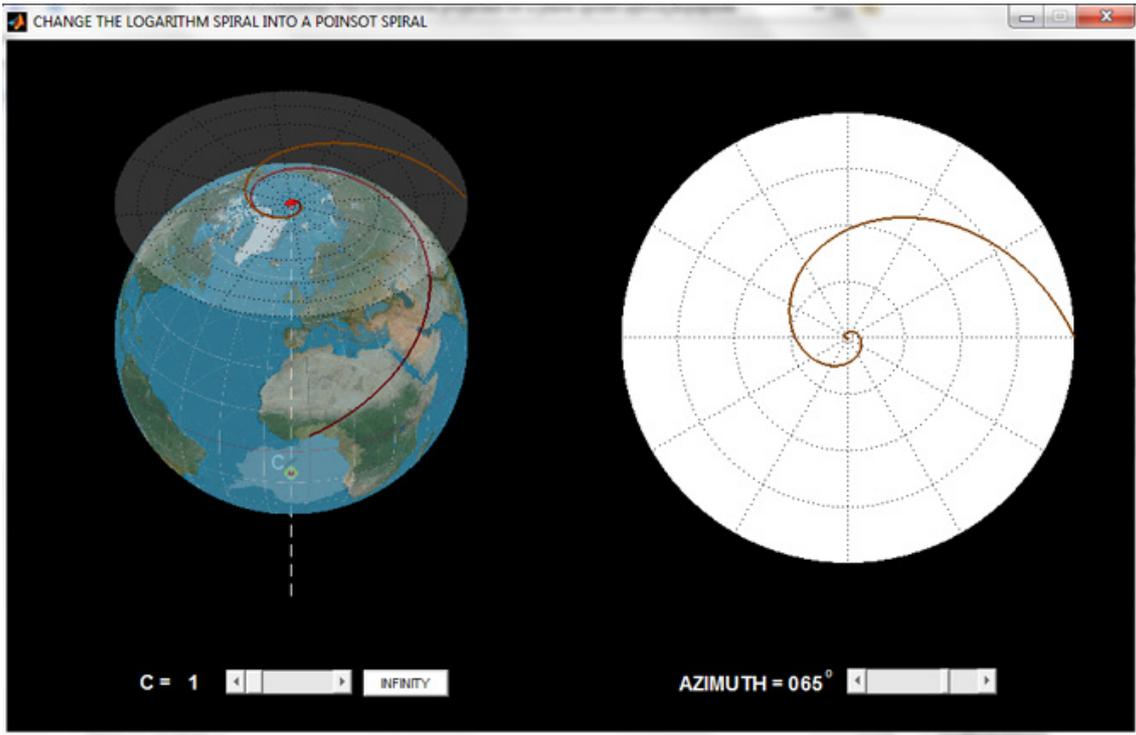
Also from the HOME the second applet compares pictures between Loxodromes and Orthodromes and the distances along them in between a DEPARTURE and ARRIVAL places, using a menu of suggested cities or just by introducing their latitudes and longitudes.



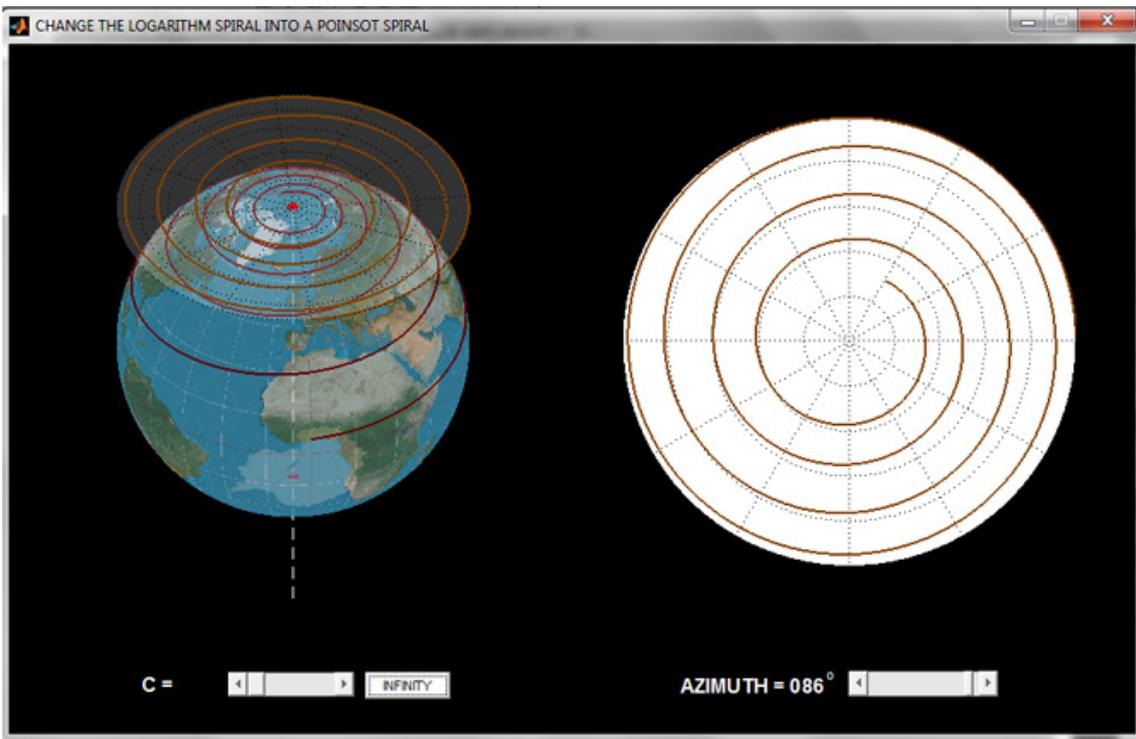
The second chapter shows THE LOXODROME PROJECTED ON A PLANE and their respective SPIRALS. It also starts by an introduction on how to change the logarithmic spiral, corresponding to the stereographic projection, into a Poincot spiral, corresponds to the orthographic projection.



From the button **Choose an Azimuth and change C [point of perspective]** the reader opens an interactive window where he can choose the Azimuth and change the point of perspective. In CHANGE THE LOGARITHMIC SPIRAL with a chosen azimuth, for instance  $65^\circ$ , as in the following window the user can obtain a deformation into a Poincot spiral.

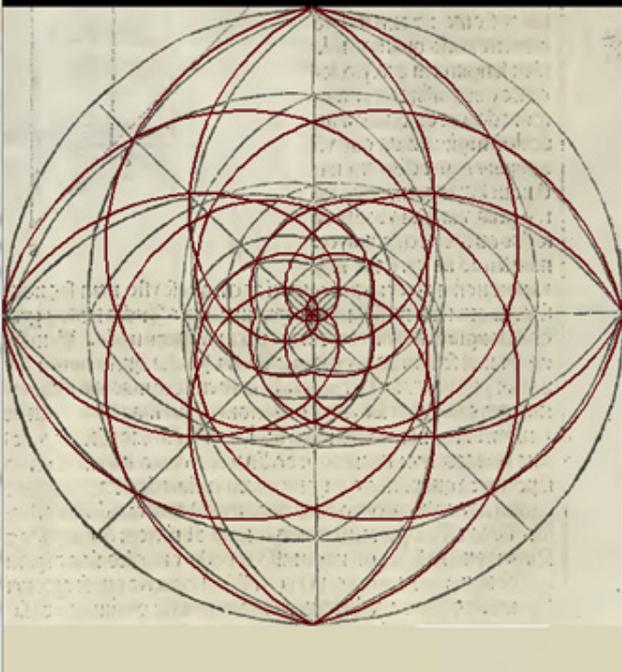


which may be illustrated with any other azimuth, as in the case of  $86^\circ$  below.



The last interactive application challenges the reader to FIND THE FOCUS OF THE ENIGMA OF THE NUNES' ROSETTE by using a slider to obtain the parameter that fits better the historical illustration.

FIND THE FOCUS OF THE ENIGMA OF NUNES' ROSETTE



In his second original "Tratado em defesam da carta de marear" (Treatise defending the nautical chart, Lisbon, 1537) Pedro Nunes drew a beautiful rosette of the projection of rhumb lines in a plane with azimuths with angles of 45° and 67.5°. The drawing was a simple schematic view of the projection but it is possible to question how Nunes would have imagined it. Use the cursor and try to find a value to fit the figure of the Nunes' Rosette.

» Find the focus of the enigma of Nunes' Rosette

Answer C = 1.8

The module concludes with a windows with the CREDITS where the frontispiece of the 1537 original book by Pedro Nunes that contains the seminal ideas developed and illustrated with the use of modern digital technology.

CREDITS



**RHUMB LINES AND SPIRALS**

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IN COLLABORATION WITH

CMAFUL

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## **TECHNICAL MANUAL**

### 1. System Requirements

Windows with a 64-bit system.

(Product tested on a 64-Bit Windows 7 system)

### 2. Installation of MATLAB Compiler Runtime (MCR)

The software was compiled with the MATLAB Compiler.

MATLAB Compiler Runtime (MCR) is necessary to execute the MATLAB application.

The MCR for the 64-Bit Windows system can be found inside the folder “Loxo.zip”

Or at the MathWorks Web site:

<http://www.mathworks.com/products/compiler/mcr/index.html>

### 3. Running the Code

The software can be found inside the folder “Loxo.zip”.

To execute just double click on the following file “Loxo.exe”.

## **INTERACTIVE PROGRAMME:**

File “Loxo.exe”

## A SHORT INTRODUCTION TO THE ORIGIN OF LOXODROMES

Navigation was one of the most important human activities in the sixteenth century. The Portuguese mathematician and cosmographer Pedro Nunes (1502 – 1578) proposed that the common art of navigation should be mathematized. Today, he is recognized as the creator of a new scientific discipline: nautical science.

Nunes became Cosmographer in 1529. Among his tasks was the management of the art of navigation. He realized that the lack of mathematical knowledge by seamen resulted in many misinterpretations and problems in navigation, so he claimed that they should be trained in mathematics. But Nunes and sea pilots lived in different conceptual realities – sea pilots were specialized practitioners, mostly trained aboard ships and their practice was a function of experience and of the application of simple procedures and rules whilst Nunes was mostly a theoretical mathematician.

Along his scientific career he published five major works. Two of them were dedicated to nautical science: his first book, generally known as the *Tratado da sphaera*, was published in Lisbon in 1537. This text consisted of translations into Portuguese of the famous *Tractatus de sphaera* by Sacrobosco, of Book I of Ptolemy's *Geography*, and of the chapters on the sun and moon in Peurbach's *Teoricæ novæ planetarum*. This edition also included two original treatises on theoretical navigation -- *Tratado que ho doutor Pêro nunez fez sobre certas duuidas de nauegação* (Treatise made by doctor Pedro Nunes about certain doubts on navigation), and *Tratado que ho doutor pêro nunez Cosmographo del Rey nosso senhor fez em defensam da carta de marear* (Treatise made by doctor Pedro Nunes, king's cosmographer, defending the nautical chart).

In these two treatises, he pioneered the use of geometrical and trigonometrical tools to solve navigation problems. This approach was uncommon in the art of navigation literature of his time, which presented mostly straightforward sets of rules, tables, mnemonics and simple introductions to the

*Sphere*. In that sense, Nunes' first texts already expressed a clear distinction between what he later termed *ars navigandi* (or common seamanship) and *ratio navigandi* (or nautical science). In the process, Nunes also presented a thorough study of the nautical chart, identifying its problems, and was the first man to develop the concept of a **rhumb line**, known today as the **loxodromic curve**.

Pedro Nunes was the first man that tried to explain mathematically the difference between a rhumb line and a great circle. In his *Treatise on some navigation doubts*, he explained the nature of rhumb lines:

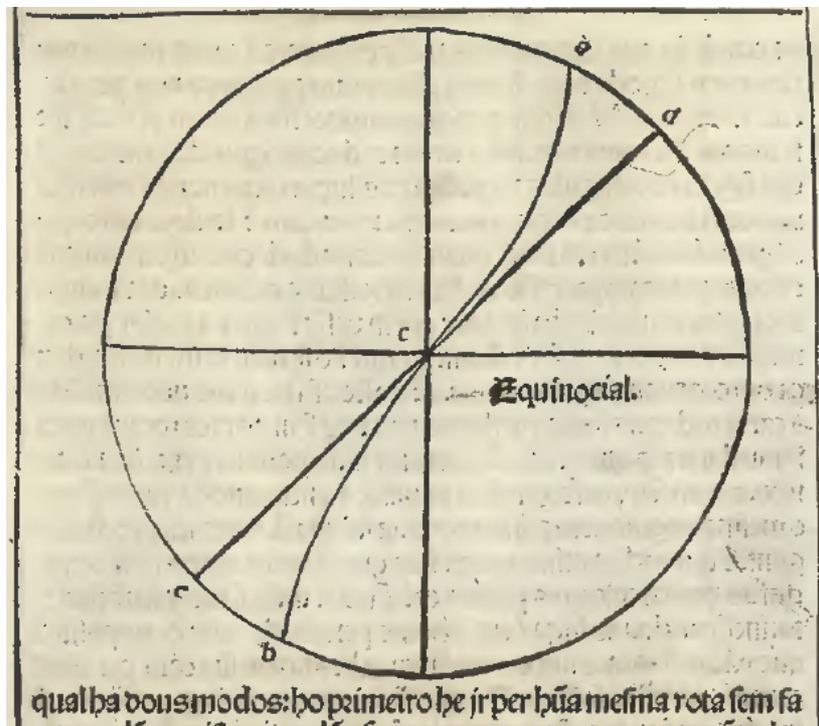


Fig 1 – The segment ab is a rhumb line and the segment cd is a great circle. *Tratado sobre certas duuidas de nauegação*, 1537.

and in the *Treatise defending the nautical chart* he draw a beautiful image of the projection of these lines in a plane:

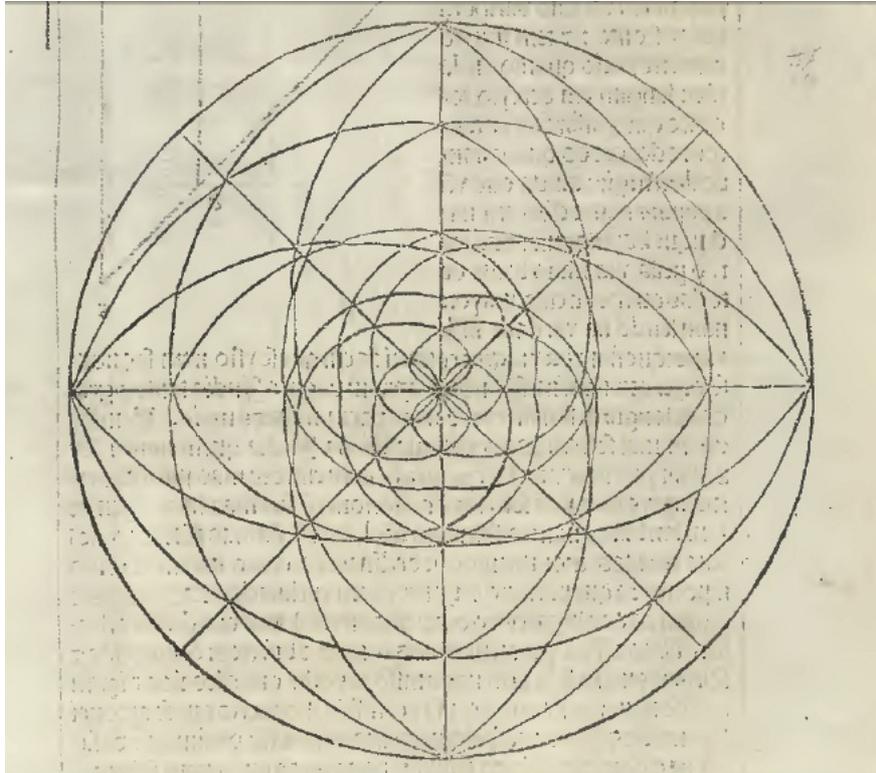


Fig 2 – Pedro Nunes' Rosette. *Tratado em defensam da carta de marear*, 1537.

Nevertheless, these mathematical concepts were not evident to most seamen of his time. In fact, the transmission of this high intellectual level knowledge to seamen was a complex process mediated by cosmographers and other mathematical practitioners.

Nunes' first treatises did not provide a method for calculating rhumb tables. In 1541, and without indicating a mathematical process for doing it, Gerard Mercator devised a globe representing rhumb lines. By this time, Nunes was already developing his mathematical theory of rhumb lines and was aware of the need to calculate rhumb tables to use in navigation, although it is not known if he already had devised a good mathematical method for doing so.

Later, in 1566 he published *Petri Nonii Salaciensis Opera*, which included his most advanced ideas on different aspects of navigation, astronomy, mechanics, and other topics. He presented a full mathematical description of rhumb lines, which included an iterative method to calculate them.

The study of rhumb lines was one of Nunes' greatest mathematical achievements. This was important both for mathematics and for navigation: it was the first time a curve on a sphere was being studied and was also a key concept for future navigators. Nunes' work on rhumb lines was much influential. Many important men of science referred to it (John Dee, Clavius, Thomas Harriot, Snell, Stevin, etc); many mathematicians contributed to the development of a theory on its stereographic projection about 150 years later by Halley.

## APPENDIX

**Great Circles** - A *great circle* is the shortest path between two points along the surface of a sphere. A great circle is the intersection of the surface with a plane passing through the center of the planet. The equator and all meridians are great circles. All great circles other than these do not have a constant azimuth, the spherical analog of slope; they cross successive meridians at different angles. The Gnomonic Projection represents arbitrary great circles as straight lines. Great circles are examples of *geodesics*. A geodesic is the shortest possible path constrained to lie on a curved surface, independent of the choice of a coordinate system.

**Rhumb Lines** - A *rhumb line* is a curve that crosses each meridian at the same angle. This curve is also referred to as a *loxodrome* (from the Greek *loxos*, slanted, biased, and *drome*, path). Although a great circle is the shortest path, it is difficult to navigate because your bearing (or *azimuth*) continuously changes as you proceed. Following a rhumb line covers more distance than following a geodesic, but it is easier to navigate. A rhumb line always spirals toward the poles, unless its azimuth is true east, west, north, or south, in which case the rhumb line closes on itself to form a parallel (small circle) or meridians. All parallels, including the equator, are rhumb lines, since they cross all meridians at 90°. All meridians are both rhumb lines and great circles.