Random processes and visual perception

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Abstract

The object of this presentation is to explore in visual terms a model of recursive thinking applied to a stochastic problem. Stochastic processes are associated to the concepts of uncertainty or chance and are a major focus of studies in various scientific disciplines such as mathematics, statistics, finance, artificial intelligence/machine learning and philosophy. Visual Art too depends on elements of uncertainly and chance. To explore the commonality of concern Science and Art and better understand stochastic processes, I used a graph theory reference model called the 'shortest route problem', and added additional elements pertaining specifically to the field of art to highlight specific phenomena of randomness in visual perception.

Introduction

The word stochastic is of Greek origin and means "pertaining to chance"[1]. It is synonymous to randomness. Randomness by nature is challenging to define and is often associated to unpredictability. Mathematicians Richard Courant and Herbert Robbins state that Mathematics offers Science both a foundation of truth and a standard of certainty based on precision and rigorous proof [2]. Greeks axiomatic geometry explores of the logic of shape, quantity and arrangement. At that time, *stochastic art* was used to differentiate arts practices such as medicine or rhetoric in which the knowledge and skill of the practitioner could not be measured simply by the results of their work. More recently, the theory of probability, to which the concept of random process is attached, opened mathematical researches to broader and more complex investigation in the area of applied mathematics, mathematical physics, mathematical biology, control theory, and engineering. In the visual arts, the perception and appreciation of an artwork depends also on random elements pertaining to light, optical alertness and various other physical and cultural parameters.

Based on this information, I selected a model used by professor Evan D Porteus for a demonstration of stochastic processes calculation [3] to illustrate mathematical reasoning in visual terms. I identified each element of the pictorial statement as objects (or numbers) and recombined them according to the scientific narrative while adding distinct components of visual communication methodology. Finally, to insure the validity of the process, I informally tested the results with colleagues from the scientific and artistic communities to highlight the common interest that joins scientific and artistic research in this field.

Random processes and mathematics

A random process or stochastic process is a collection of random variables defined on an underlying probability space. Much of the mathematics of stochastic processes was developed in the context of studying Brownian motion [4]. Brownian motion described the physical trajectories of pollen grains suspended in water. In 1905 Albert Einstein, using a probabilistic model, provided a satisfactory explanation of the Brownian motion. From 1930 to 1960 J. L. Doob and Kolmogorov, transformed of the study of probability to a mathematical discipline and set the stage for major developments in the theory of continuous parameter stochastic processes. Probability is mathematics, Doob clearly stated in the preface of his 1953 book 'Stochastic processes' [5]. More recently, Wendelin Werner's research accomplishments in the field of probability led to his being awarded a Fields Medal in 2006.

Every non-mathematical probabilistic assertion suggests a mathematical counterpart that sharpens the formulation of the non-mathematical assertion and may also have independent mathematical interest [6]. To illustrate the relationship between mathematics and the visualization of a stochastic process, I selected a series of templates used by Dr E. Porteus for his demonstration on model optimization. In this example, Dr Porteus approaches random processes from the recursive perspective and decomposes a complex problem into a series of smaller problems, after the Bellman principle called dynamic programming. The demonstration breaks the problem into two parts. First a study of a recursive model, based on a plan of 4 horizontal and 4 vertical squares. Next, a similar surface incorporates elements of uncertainty represented by circles. Squares nodes are called decision nodes; circle nodes are named chance nodes. (Fig.1).

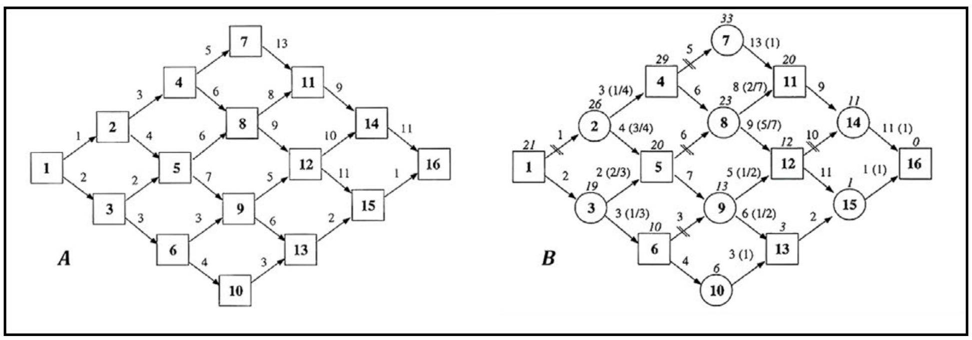

Figure 1: *The shortest route problem. A) The recursive problem B) the stochastic problem solved*

The shortest route problem visualization

The following is a succession of steps I took to convert Dr Porteus' templates into a visual statement. The models are fairly easy to compose in a graphic editor software. I recreated the 4 succeeding templates of the demonstration in a vector-based program to insure the line sharpness and clear definition of each object.

I initially worked from a simple black and white design to focus on object placement and dynamic of shapes. I created 3 different copies of the same pattern in 3 different sizes to emphasize the recursive aspect of the demonstration. I gave each surface a distinct identity defined by light intensity. I assigned each set of square/circles objects various elements of shades and texture to differentiate them from each other. I also slightly twirled the surface of the main board at a 30° angle on the 'problem solved' templates to emphasize the need to approach the problem in terms of arc rather than straight line to calculate the shortest route.

I transferred the design into a pixel-based environment to add color to the design. Rather than use random color, I selected a specific palette based on Korean traditional color to work from, because of its unique and strong identity in terms of primaries and light intensity. Korean color symbolism is based on five elements: blue, white, red, black and yellow. Their combination reflect the traditional principle of positive and negative, light and dark. It also corresponds to the four points of the compass and the center, as well as the five elements of the weather (cold, warmth, wind, dryness and humidity. [7] The Shin, Westland survey conducted for the school of design, of the University of Leeds by the Changwon National University [Fig. 2] helped me determine the spatial positioning of each color and their dynamic in the overall composition.

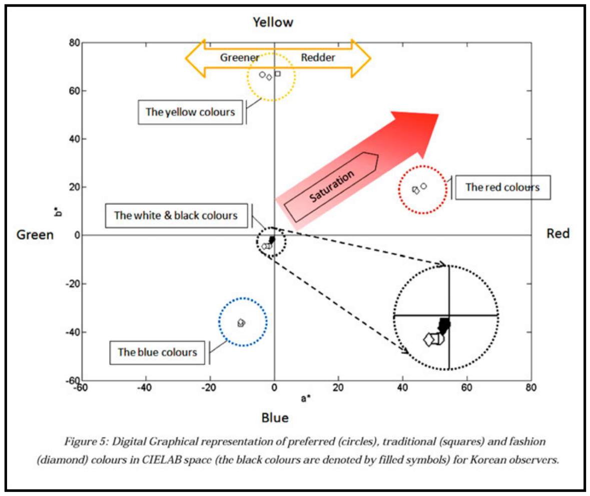

Figure 5: Digital Graphical representation of preferred (circles), traditional (squares) and fashion (diamond) colours in CIELAB space (the black colours are denoted by filled symbols) for Korean observers.

Figure 2. *Shin-Westland - CIELAB spatial color perception survey*

Randomness and visual perception.

To insert an additional element of randomness in the visual statement, I used a variation of an after-image effect first discovered by German scientist L. Hermann in 1870 that highlight the inhibition that neighboring neurons in brain pathways have on each other (Fig. 3).

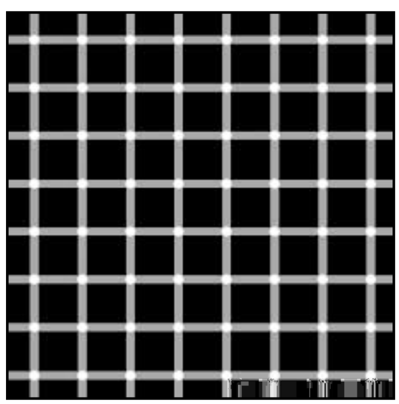

Figure 3. *The Hermann grid.*

I increased the size of square #1 (left-center line) of the final model "shortest route problem solved" of 5 points and slightly changed the opacity of color to prompt the eye to subjectively assert what is the minimum distance solution of the stochastic problem. The ensuing design induces the eye to fill black space with white dots and cover white dots black. This effect varies depending on various parameters having to do with visual perception, alertness of the nervous system as well as in this specific case, inference of elements of color size and shape (Fig. 4).

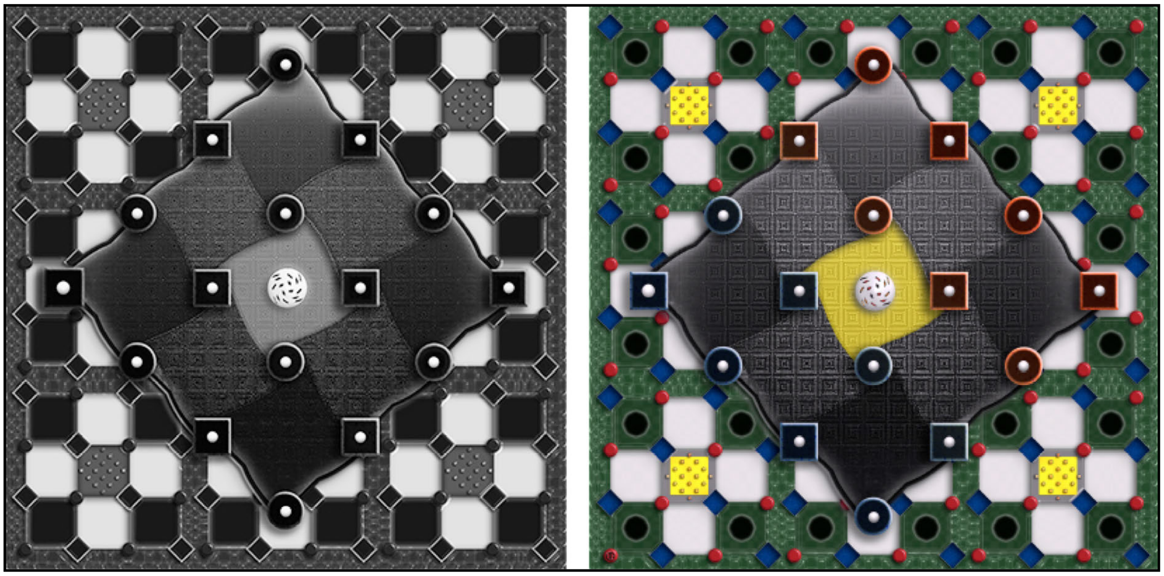

Figure 4. *The 'Shortest route problem solved'. A) B&W. B) Color.*

Finally I conducted an informal test with several colleagues, mathematicians, architects and art historians. The consensus was that, once the purpose explained, it made the problem solving challenging, intriguing but easier and fun to conduct. As an artwork, several mentioned a similarity to kinetic art - which indeed deals too with issues of light, color and random after images effect.

Conclusion

These researches were made in a very intuitive and empirical manner. Using a stochastic mathematics methodology would help to create a more exact and effective statement, in particular regarding the best possible canvas size to create the after-effect or the optimal distance from the composition to validate the random effect. Applications of mathematics in the field of random processes have emerged across the entire landscape of natural, behavioral, and social sciences, from medical technology to economic planning (input/output models of economic behavior), from genetics to geology (locating oil reserves). More academic and scientific studies will insure a better comprehension of the process. It will also help artists get a better handle on the tools they use to convey their statement [8]and provide the viewer with a distinct esthetic experience.

References

[1] http://mathworld.wolfram.com/Random.html (as of Dec.28, 2013)
[2] Richard Courant and Herbert Robbins. *What Is Mathematics?* Second Edition Oxford University Press, 1996.
[3] Evan L. Porteus. *Foundations of Stochastic Inventory Theory.* Stanford University Press, 2002.
[4] M. Scott. *Applied Stochastic Processes in Science and Engineering.* Department of Applied Mathematics, University of Waterloo, ON. 2013.
[5] J. L. Doob. *Stochastic Processes.* John Wiley & Sons. 1953.
[6] J. Laurie Snell. *A conversation with Joe Doob*. Statistical Science, Vol. 12, No. 4. November 1997.
[7] Meong Jin Shin, Stephen Westland, Edel M Moore and Vien Cheung. *Colour preferences for traditional Korean colours.* Journal of the International Colour Association. 2012.
[8] Jean Constant *'Stochastic Art'*. http://www.hermay.org/jconstant/stochasticart/ (as of March 15, 2014)