Twizzle Torus

The Twizzle Torus is an annular surface with a constant mean curvature in the three-dimensional sphere, a space curved in itself. To make it visible it must first be projected into our flat space. Luckily, this is enabled such that basic shape features can be maintained. In the three-dimensional sphere where it belongs it has a screw symmetry which can still be imagined during projection.

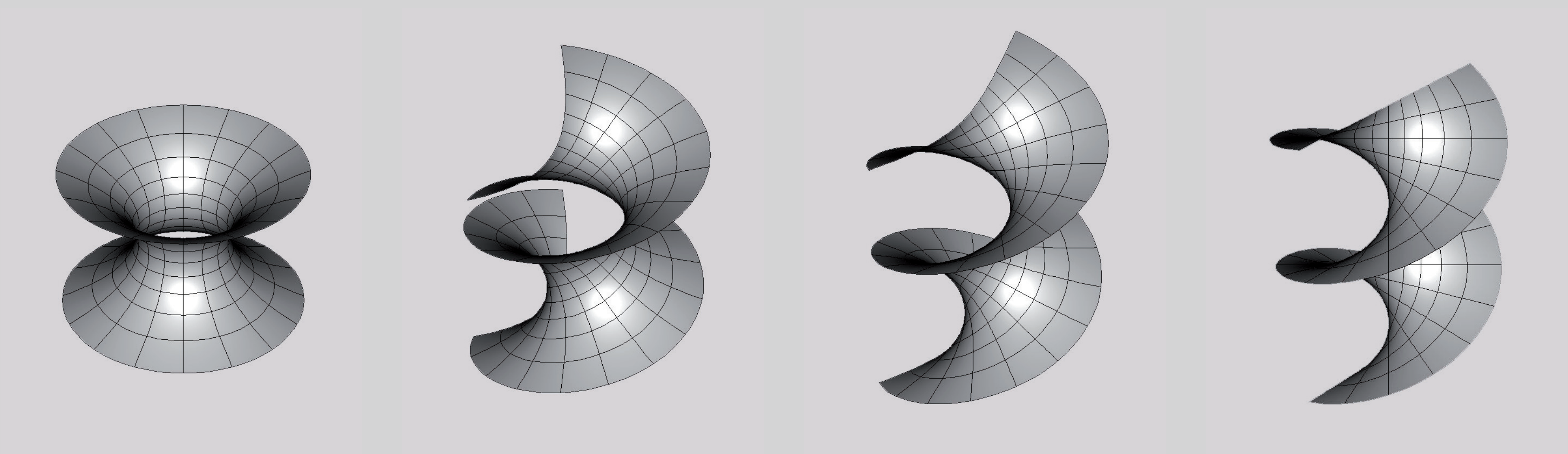
The Twizzle Torus is only a comparatively simple example in an endless hierarchy of increasingly complex annular surfaces with similar curvature features.

The surface was developed by Nicholas Schmitt (Geometrie-Werkstatt Tübingen) and he has designed and calculated the image using XLab software.

Design: Nicholas Schmitt

Discrete minimal surface

Minimal surfaces are a classical subject matter of differential geometry. They are surfaces whose mean curvature vanishes everywhere. The best known among this surface class are the Catenoid and the Helicoid.



One of the many interesting features of minimal surfaces is the existence of an associated family. This means that the surfaces are deformed in such a way that they remain minimal (in technical terms, they are isometrical to each other and corresponding tangent planes are parallel). Helicoid and Catenoid belong to the same associated family. As a result, they can be deformed into one another and all the surfaces in between are also minimal.

The image shows a discretization of the minimal surface half way between Catenoid and Helicoid. It is composed of spheres and circles touching each other at their points of contact. There is also an associated family for these discrete minimal surfaces and both the radii of corresponding spheres and the positions of corresponding circle disks are equal.

Design: Tim Hoffmann

Boy surface

The Boy surface is a non-orientable surface that is one possible parametrization of the surface obtained by sewing a Möbius strip to the edge of a disk. It was found by Werner Boy in 1901 and it is a model of the projective plane without singularities (but with self-intersections). The Mathematisches Forschungsinstitut Oberwolfach has a large model of a Boy surface outside the entrance, constructed and donated by Mercedes-Benz in 1991.

The version shown here is characterized by its mean curvature being as small as possible. It has "no unnecessary bumps" in this sense. Here, you see the most "beautiful" possible realization of a Boy surface in a mathematically precise sense. This is a parametrization of a Boy surface by Robert Bryant and Robert Kusner.

The image is based on a bullet panorama which Paul Debevec generated from photos of a church in San Francisco. The scene was compiled in jReality, the image itself was calculated with Sunflow. Supported by DFG-Research Centre Matheon.

Design: Ulrich Pinkall

Björling surface

Minimal surfaces are surfaces which have the same curvature features as physical soap films. The construction of minimal surfaces with given features is a classical subject of differential geometry. In 1844, E. G. Björling showed that for each sufficiently benign space curve a narrow minimal surface strip can be found which contains the curve. Furthermore, it can even be specified how the strip shall twist around the curve.

The surface shown here is generated with the basic curve as a Helix along which the strip is twisted at constant speed.

The formulas of this particular Björling surface come from Matthias Weber. The image is based on a computer generated landscape compiled using Terragen by Simon O'Callaghan. The scene was developed with jReality; the image itself was calculated by Sunflow. Supported by DFG-Research Centre Matheon.

Design: Ulrich Pinkall

Tetranoid

The Tetranoid belongs to the class of surfaces which have the same curvature features as physical soap bubbles. Mathematically, the Tetranoid has a “constant mean curvature”, as the four “legs“ of the Tetranoid are actually never ending.

The existence of the Tetranoid (like the existence of similar surfaces with any symmetry based on the Platonic solids) was proved by Nicholas Schmitt. He also calculated the surface.

The image is based on a bullet panorama which Paul Bourke generated from photos. The scene was compiled in jReality, the image itself was calculated with Sunflow. Supported by DFG-Research Centre Matheon.

Design: Ulrich Pinkall

Helicoid with handles

One of the best known minimal surfaces is the Helicoid which is a shape like a spiral staircase or car park ramp. It is, indeed, possible to connect different sheets of the Helicoid with each other without destroying the minimal surface feature or making the surface intersect itself.

This connecting piece is called a “handle“ in mathematical terminology. Depending on which storey you are on, such a handle looks like a hole in the floor or ceiling, or even like a column which connects floor and ceiling of a storey.

The surface shown is a Helicoid with two handles and was found and calculated by Markus Schmies. The image is based on a computer-generated landscape compiled using Terragen by Simon O’Callaghan. The scene was developed with jReality, the image itself was calculated by Sunflow. Supported by DFG-Research Centre Matheon.

Design: Ulrich Pinkall

Bursting Nodoid

The bursting Nodoid is a special surface with constant mean curvature. Such a surface can be imagined as a boundary surface between two liquids or gases at different pressure such as a soap bubble enclosing a certain air volume. In contrast to physical soap bubbles mathematical soap bubbles are allowed to intersect themselves.

You have to imagine that all five extensions which you see emanating from the surface continue without end. And a balance of forces is achieved: The four undulating tubes draw the upper end down and thus keep the middle column pressing upward in balance.

The surface was developed by Nicholas Schmitt (Geometrie-Werkstatt Tübingen) and he also designed and calculated the image using XLab software.

Design: Nicholas Schmitt