A Victorian Age Proof of the Four Color Theorem

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Abstract. In this paper we have given an algorithmic proof of the four color theorem which is based only on the coloring faces (regions) of a cubic planar maps. Our algorithmic proof has been given in three steps. The first two steps are the maximal mono-chromatic and then maximal dichromatic coloring of the faces in such a way that the resulting uncolored (white) regions of the incomplete two-colored map induce no odd-cycles so that in the (final) third step four coloring of the map has been obtained almost trivially.

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INTRODUCTION

Four color map coloring problem is to color regions of a (normal) map $M$ with at most four colors so that neighbor regions (countries) would have receive different colors. This simple problem posed and conjectured to be true for all maps by Guthrie in 1852 [1],[17]. Its correct proof was first given in 1976 and repeated several times by the same method by the help of a computer [2]-[5]. The author has been proposed two non-computer proofs, one on vertex coloring of the maximal planar graphs and the other on edge three coloring of the bridgeless cubic planar graphs, of the four color theorem all based on spiral chains [6],[7],[8].

In this paper we will give another proof of the four color theorem based on step-wise mono-chromatic coloring, two coloring and then four coloring of any given normal map $M$, i.e., four coloring of the faces (regions) of any cubic planar graphs (maps). That is the proof is rely on the elements of the cubic planar maps and a coloring strategy. Therefore our proof suits more with the mathematics of the Victorian age [16],[17] in which the four color problem arose. Of course any four coloring of $G$ induces edge disjoint two bipartite graphs but not necessarily connected. We have also suggest surveys on the early developments of the four color problem by Saaty [9] and Mitchem [10].

A NEW PROOF OF THE FOUR COLOR MAP THEOREM

A more courageous title of this section would be "How to create a four colored world in three steps?" It is well-known and without doubt that four color theorem is true. What are the reasons for a lengthy existing proofs by the use of a computer? One answer would be going to the long way which has been forced by the false Kempe’s "proof", see
for example Birkhoff’s reducibility of double $C_5$ (actually overlapped 4 cycles of length 5)[11]. Another answer would be over looking difficulties of the planar three colorability problem in the light of Grotzsch and Heawood’s theorems [12],[13]. Starting point of the classic tedious proofs of the four color theorem is based on Kempe’s failure of the reducibility of the pentagon case. That is the correct proof gives up on pentagons and turns to larger reducible configurations for which Kempe’s argument is sound [1]-[5]. The first such configuration which has ring-size 6, was discovered by Birkhoff [11] known as Birkhoff’s diamond. The Birkhoff’s diamond is also inspirational starting point in our spiral ordering and maximal mono-chromatic and maximal dichromatic coloring of the maps which leads to an non-computer proof. In this section we will be giving a new proof of the four color map theorem in which we have implicitly by pass the three-coloring problem of planar graphs within the constructive proof [8].

In fact our algorithmic proof implies the following theorem without relying on the four color theorem [14],[15]:

**Theorem 1.** Every planar graph can be decomposed into the edge disjoint union of two bipartite graphs.

Let us denote by $M$ an normal map with $n + 1$ regions, where $(n + 1)$th region $r_{n+1}$ is the outer-region of $M$. Without loss of generality we may further assume that $M$ is digon-free (two-side region) and triangle-free (three-side region). Since if the map has a digon or triangle we shrink it to a point. Since then we can four color the resulting map and put back digons and triangles; it’s surrounded by at most three colors. so there is a spare color to color the digon or triangle, as required. $M$ can be equivalently represented by a cubic planar graph $G_c(M) = (V_c, E_c)$, where $V_c$ is the set of vertices associated with the crossing of pairwise three neighbor regions, and $E_c$ is the set of edges in the form of Jordan curve associate with the boarder of two neighbor regions between two vertices.

In order to make the map-coloring algorithm more visible and meaningful let us define the four-color set as $C = \{B, G, dB, lB\}$, where
- $B$ denotes brown color and when it is assigned on to the white background color the corresponding region becomes a "high-land".
- $G$ denotes green color and when it is assigned on to the white background color the corresponding region becomes a "low-land".
- $dB$ denotes dark-blue color and when it is assigned on to the white background color the corresponding region becomes a "deep sea".
- $lB$ denotes light-blue color and when it is assigned on to the white background color the corresponding region becomes a "shallow-sea".

Initially the given map colored all by background color white and at the end of the coloring algorithm (in three steps) it will be colored by the colors $C$ and no white color remains on the map. Clearly we will show that this is always possible for any map $M$.

By $M(B)$ we denote a map in which maximal number of its regions colored by $B$ (mono-chromatic coloring) where the term maximal means that any additional brown region (high-land) results color conflict and all the remaining regions are background-
color white. Similarly by \( M(B,G) \) we denote a map obtained from \( M(B) \) in which maximal number of its white regions colored by \( G \). Hence \( M(B,G) \) is an maximal two-coloring of \( M \).

**Definition 1.** In a mono-chromatic coloring of map \( M(B) \) if an vertex \( v \) is not incident to any brown colored region then \( v \) is called unwanted-spot or simply a spot. Furthermore if the map \( M(B) \) is spot-free then the map \( M(B) \) is called clean map.

**Definition 2.** Spiraling of a map \( M \) is a process of ordering and labeling the faces (regions), starting from the outer-region \( r_{n+1} \) and selecting always outer next region \( r_i \) neighbor to the previous region \( r_{i+1} \) in the form of a spiral.

Note that depending on the adjacency of the regions of the map \( M \) we may have several spirals but the ordering of the regions is uniquely determined by the initial region and next one with the direction selected e.g., clockwise or counter clockwise. Start with the outer face and label it \( r_{n+1} \). Then, draw a curve from a point in \( r_{n+1} \), crossing an edge into an adjacent face. Label that face \( r_n \). The faces adjacent to \( r_n \) (apart from \( r_{n+1} \)) are ordered clockwise; choose the first (i.e. leftmost) such face, cross into it, and label it \( r_{n-1} \). Proceed in this fashion, always crossing into the leftmost available face that has not been visited already. At some stage one will be unable to proceed. If all faces have been visited, then the spiral chain \( S_1 = \{ r_{n+1}, ..., r_1 \} \) is the spiral ordering. Otherwise, start at the closest face to the last face of \( S_1 \) and produce a new chain \( S_2 \). And so on till all faces are in some chain. Similar definition has been given for maximal and cubic planar graphs in [6],[7]. For an illustration spiraling see the nested three spirals \( S_1, S_2 \) and \( S_3 \) shown in Figure 4.

**The map coloring algorithm**

Main feature of the coloring algorithm is the use of each of the four colors one-by-one and preparing the conditions satisfied for the next step.

**Step 1. Maximal mono-chromatic coloring of high-lands map \( M(B) \).**
Let \( S = \{ r_{n+1}, r_n, r_{n-1}, ..., r_1 \} \) be the spiral ordering of the faces of map \( M \). Color outer-face \( r_{n+1} \) of \( M \) with \( B \). Along the spiral \( S \) color next white region \( r_i \in S \) with \( B \) by the following rule:

(i) All the first neighborhood of the region \( r_i \) remain in white (uncolored).
(ii) If any white region \( r_j, j > i \) is colored, that is \( c(r_j) = B \) then a color-conflicts arises.
(iii) At least one of the second neighborhood region \( r_i \) with maximum number of sides would be colored by \( B \). Note that all vertices of the map \( M \) can be considered as spots since it is cubic planar and no region colored in brown.

Using (i)-(iii) and spiraling \( S \) the maximal mono-chromatic set of \( k \) regions can be obtained. Let us call the map \( M \) after the coloring as \( M(B) \). Let us also denote the spots of \( M(B) \) with a set \( P = \{ p_1, p_2, p_3, ..., p_k \} \) where \( k < n \). That is \( P \) is the set of
triply neighbor white regions of the map $M(B)$ where some of the white regions may be overlapped.
The output of the step 1 is simply maximal disjoint of highland islands all colored in brown.

**Step 2. Maximal dichromatic coloring of high-low-lands map $M(B, G)$**.

We use the same spiraling $S$ of the map $M(B)$. While assigning color green $G$ to a white region consider the following two conditions:

(i) Along the spiral ordering when assigning green color to white regions give priority to the white-region which has maximum number of spot vertices in $M(B)$;

(ii) Do not create any $(B, G)$-ring $R(B,G)$ which contains an inside odd white-ring $R(W)$ and do not leave any spot vertices.

We have also the following simple property of $M(B)$.

**Lemma 2.** The spots of the triply neighbor white regions of the map $M(B)$ cannot induces a cycle.

**Proof.** Let us assume that a region $r$ colored by $B$ has been surrounded by an cycle of spot vertices. Hence regions in the second neighborhood must be also all white. But (iii) we have colored at least one of the region in the second neighborhood in $B$ and that breaks the cycle of the spots into a path.

As it has been seen that Step 1 is rather straight forward and map $M(B)$ can easily be obtained for any $M$. Assuming the maximal mono-chromatic coloring of $M(B)$ as a base, it is not such an easy task to obtain dichromatic map $M(B, G)$. In the next step we will give the details and proofs that starting from mono-chromatic $M(B)$ it possible to two-coloring of $M(B, G)$ with a set of properties that satisfies four colorability of the whole map. That is we will show that by assigning color green (color for low-land) to the some of the white regions of $M(B)$ we obtain maximal dichromatic coloring of $M(B,G)$ without any spots, without any even $(B, G)$-ring and without odd any $W$-ring (white-rings in $M(B,G)$).

**Lemma 3.** Mono-chromatic (green) spiral-chain coloring of the white regions of the map $M(B)$ results in a spot-free map $M(B,G)$.

**Proof.** If a spot-vertex remain in $M(B,G)$ it would be one of the bad configurations illustrated in Figures 1. But this bad configuration can only occur when green color assigned without considering the maximum number of spots of the white region. However this has been protected by Step 2 (i) in the algorithm.

**Lemma 4.** The maximal di-chromatic map $M(B, G)$ obtained by the algorithm has no odd-white-ring of length 3.

**Proof.** Lemma follows since we assumed that the cubic planar map $M$ has no triangle and $M(B, G)$ is spot-free.
How to avoid odd rings in $M(B)$?

The maximal mono-chromatic coloring $c$ of the map $M(B)$ shown in Figure 2 has been provided as an possible counter-example to the map coloring algorithm. One can easily see that the maximal mono-chromatic coloring of $M(B)$ can never be obtained by any spiral ordering of the faces of $M$. Simply not all 5 square regions e.g., the outerface and the four colored faces other than the pentagons $P$ and $Q$, can be selected by the Step 1(iii). Note that even if odd rings created by the map coloring algorithm we can assign provisionally color green to the critical faces that is common in maximum number of white odd-rings. In Figure 2 the critical faces are $X = b_8$ and $Y = b_7$ and coloring one of these faces by green breaks all odd-rings. This will be discussed in detail in the next sub-section.

Along the spiral ordering of $B$-coloring (brown color) of the regions of $M$ if we do not give priority to the region with the maximum number of spots (Step 1(iii)) i.e., region with maximum sides, then there exists certain "counter-examples" that spiral coloring algorithm requires the fifth color. Simplest map with this property is shown in Figure 3(a) together with a maximal dichromatic coloring of $M(B,G)$ when the face $R$ is assumed as the outerface. This is possible since the map can be redrawn so that the chosen region is the outside. The resulting spiral chains is shown in Figure 3(b). Since $M(B,G)$ of Figure 3(a) has two odd-white rings of length 5, it cannot be extended to four coloring. It is
FIGURE 2. Odd rings in a maximal mono-chromatic coloring of a map $M(B)$.

It is straightforward that pentagons $P, Q$ must receive the same color in any 4-coloring, since any attempt with $c(P) \neq c(Q)$ results in use of fifth color on the region neighbor to the outerface. This can only be resolved by the use of appropriate Kempe’s switching that results in $c'(P) = c'(Q)$. On the otherhand as in Figure 3(b) if in $B$-coloring we select region $X$ with maximum number of sides instead of brown square of Figure 3(a), the coloring of $M(B,G)$ ends up without odd-white cycles.

To show that spiral ordering uses no more than four colors, consider $B$-coloring of Figure 3(c). Spiral ordering is shown in red-dashed curve. Outerface is colored by $B$ as usual. The second region colored by $B$ is the square region on the left side of the map. However Step 3 (iii) has not selected next square (colored in light blue) since the neighbor region $X$ has eight sides. The last region to be colored by $B$ is $R$ which is the first region of Figure 3(b). Now Step 2 of $G$-coloring chooses regions $Y$ and the left and right regions since they all have 8 sides and removes the 4 spots. Clearly $M(B,G)$ has no odd-white rings and can easily be extended to a four coloring.

**Blocking big-odd-white cycles in $M(B,G)$**

From the above discussion it is possible to claim that map coloring algorithm would not generate $M(B,G)$ with odd-white rings (cycles). It is difficult to prove this claim, in the absence of control of detecting odd-white cycle of size greater than 3 in the algorithm. We will give a simple dynamic binary labeling algorithm that maintaining the parities of all white rings till the end of the map coloring algorithm. Let $M(B)$ be the maximal mono-chromatic coloring of $M$. Let $R = \{r_{n+1}, r_n, ..., r_1\}$ be the set of regions of $M(B)$. Clearly $c(r_{n+1}) = B$ but we do not certain about other regions (islands) in color $B$. Any attempt to color a white-region with $B$ results a color conflict in $M(B)$. Let $M_o(B,G)$ be the set of all maximal dichromatic maps that have at least one odd-white cycles of length greater than 3. Call these odd cycles in $M_o(B,G)$ as big-odd-white (simply odd-cycle) cycles.

Define an binary labeling $f$ of an region $r_i \in R$ as follows:
$f(r_i) = \begin{cases} 1 & \text{if } |r_i| \equiv 1 (mod\ 2) \\ 0 & \text{if } |r_i| \equiv 0 (mod\ 2) \end{cases}$.

Beginning of the Step 2 ($i = 1$) we have the set $F_i$ of binary labels

$F_i = F_i(W) \cup F_i(B) \cup F_i(G)$

Initially $F_i(B, G) = \emptyset$, (for $i = 1$) where $F_i(B, G)$ is the set of all binary labels corresponding to the disjoint sub-maps $M_i(B, G)$ computed by

$f_i(M(B, G)) = \sum_{r_j \in M_i(B, G)} f(r_j) (mod\ 2)$

and $F_i(W) = \{f(r_j) | \text{if } c(r_j) = W\}$, $F_i(B) = \{f(r_j) | \text{if } c(r_j) = B\}$ and $F_i(G) = \{f(r_j) | \text{if } c(r_j) = G\}$.

It is apparent that at each color "green" assignment of Step 2 the size of the set $F_i(B, G)$ increases at most by one and priority is given to the white region with maximum spots and with the even sizes. At some step $k$ we have obtained
FIGURE 4. The Haken and Appel’s map. This map has been taken from Ed Pegg Jr’s mathpuzzle.com/4Dec2001.htm. Haken and Appel needed a computer to 4-color the following hardest-case map, which has been presented in a slightly different form. Spiral ordering of the regions are also shown and denoted by $S_1, S_2$ and $S_3$.

\[ F_i(B, G) = \{ f(M_1), f(M_2), \ldots, f(M_k) \} \]

which corresponds to all disjoint sub-maps of the maximal di-chromatic coloring of $M(B, G)$. If all $f(M_i) = 0, i = 1, 2, \ldots, k$ then $M(B, G)$ is white-odd ring free.

**Definition 3.** Let $M(B, G)$ be a maximal dichromatic map. A region $r_j$ colored by $B$ or $G$ is called an island if it is surrounded by the white regions. Similarly we can define a continent as a connected regions of sub-map $M_k(B, G) = \{r_1, r_2, \ldots, r_k\}$ colored by $B$ and $G$ if it is surrounded by the white regions.

Let $M_s(B, G)$ be a maximal connected two colored sub-map of $M(B, G)$. Then the white ring $R(M_s(B, G))$ that surrounds $M_s(B, G)$ is an white-odd ring if and only if the number of odd faces (including induced white faces) in $M_s(B, G)$ is odd. That is white ring $R(M_s(B, G))$ is odd iff $f(M_s(B, G)) \equiv 1 (mod 2)$.

This property is quite useful when we are testing whether or not an odd-ring surrounds a maximal connected sub-map in $M(B, G)$.

We will show that by using simple transformations (Kempe’s chain switching) on the odd cycles it is possible to remove all odd cycles from $M_s(B, G)$. Again, this will be illustrated on the "counter-example" map $M(B, G)$ given in Figure 2(a) in two ways:

(a) Each individual odd ring blocked separately.

Consider the sub-map $M_s(B, G)$ containing green pentagon $Q$. Use Kempe switching for the regions of $M_s(B, G)$. Now the upper and lower white regions neighbor to the
outerface are all surrounded by brown-white regions. Therefore we can assign color green to these regions (denoted by $b_1$ and $b_2$) and block the odd ring that surrounds $M_s'(B, G)$. Now consider the sub-map $M_{s''}(B, G)$ containing the pentagon $P$. The odd ring that surrounds $M_{s''}(B, G)$ can be easily blocked by joining the two islands (brown square regions) by assigning green color to the regions denoted by $b_3$ and $b_4$ and changing the green region of $M_{s''}(B, G)$ into white color.

(b) **Blocking the two overlapped odd rings.**

Use Kempe-switching for the regions of the sub-map $M_{s'}(B, G)$ and move green regions to the white region neighbor to the pentagon $P$. Now the white regions denoted by $X$ and $Y$ which are common for both odd-rings are surrounded by brown and white regions. Therefore we can assign color green ($b_1$) to either region $X$ or $Y$ and block both odd rings.

From the spiral coloring algorithm as well as blocking big-odd-white cycles we observe that if the number of disjoint sub-maps $M_s(B, G)$ is minimum then there is no odd white cycles in the map $M$.

The following lemma is useful and confirms the discussions above in general.

**Lemma 5.** Let $M(B, G)$ be a maximal dichromatic map which has minimum number of disjoint sub-maps $M_s(B, G)$. Then there is no odd-island in $M(B, G)$.

**Proof.** Since the regions neighbor to odd-island must be visited by the spiral ordering if the odd-island colored by $B$ (brown) in the first round of spiral coloring at least one neighbor region must be colored by $G$ (green) in the second round of spiral coloring.
FIGURE 6. Maximal two coloring of high-land (brown(dark-gray)) and low-land (green(light-gray)) regions. Green coloring starts from the upper white region (can be started any white region adjacent to outer region). Trace of green regions form a spiraling in the clockwise direction and at each step at least one "circle" of Figure 5 is vanished by the assignment of the green color to a white region. By red-dashed curves we have shown five even white-rings (even-cycles) around the brown-green (high-lowland) islands. The rest of white regions induce an acyclic graph. The trees (blue) show (B,W)-Kempe chain started from region Y to change the color white of the region X into brown and (B,W)-Kempe chains started from regions Y' and Y'' to change white X' into brown. This is the only transformation that will be applied to break odd-white ring in M(B,G).

The following lemma is important in establishing the non-computer proof of the four color theorem. Enables one to break any odd white ring in the dichromatic coloring of the map by re-arranging green colored regions around the brown regions.

Lemma 6. Let M(B,G) be a maximal di-chromatic map without spots. Let R(W) be a white odd-ring of length greater than 3 in M(B,G). Then M(B,G) can be re-colored to make M'(B,G) white odd-ring R(W) free.

Proof. In fact we will show that for a given maximal di-chromatic coloring of M(B,G) by the use of certain Kempe-chains switching it is possible to re-color any selected white region r(w) (odd or even sizes) into green G or brown B color. Let r(w) ∈ R(W) where R(W) is denoting white odd ring of size greater than 3. Simplest case is when region r(w) is an square S. Let \{r_1, r_2, r_3, r_4\} be the regions neighbor to S. The two non-adjacent regions must be white, say c(r_2) = c(r_4) = W and if the other two non-adjacent regions are green c(r_1) = c(r_3) = G then color c(r(w)) = B or if c(r_1) = c(r_3) = B
then color \( c(r(w)) = G \). If \( c(r_1) = B \) and \( c(r_3) = G \) then we use Kempe switching to the \((B,G)\)-Kempe chain where \( r_1 \) is the first region in the chain. So we can assign \( c(r(w)) = B \). Or we use Kempe switching for the \((G,B)\)-Kempe chain, where \( r_3 \) is the first region in the chain. So we can assign \( c(r(w)) = G \). Hence the case of white square \( S \) is settled. Now consider the case when \( |r(w)| \geq 5 \) which is a bit different than the above. Let \( r(w) \) be a pentagon \( P \) with neighbor regions \( \{r_1, r_2, r_3, r_4, r_5\} \). Clearly if \( c(r_1) = c(r_3) = G \) and \( c(r_2) = c(r_4) = c(r_5) = W \) then we assign \( c(P) = B \). So we assume \( c(r_4) = B \) (or \( c(r_5) = B \)) but not \( c(r_2) = G \) since the white pentagon \( P \in R(W) \). Now consider a \((B,W)\)-Kempe chain in \( M(B,G) \) starting from the region \( r_4 \) such that:

(i) \((B,W)\)-Kempe chain ends up with a region colored by \( B \) surrounded by all \( W \) and \( G \) regions or ends up with a region colored \( W \) surrounded by all \( W \) and \( G \) regions and

(ii) when Kempe switching applies to \((B,W)\)-Kempe chain no spots have been generated.

Hence after the Kempe switching of \((B,W)\)-Kempe chain we have \( c(r_1) = c(r_3) = G \) and \( c(r_2) = c(r_4) = c(r_5) = W \) and we assign again \( c(P) = B \). Now if \( |r(w)| \geq 5 \) we apply one by one \((B,W)\)-Kempe chain switching for each brown neighbor region of \( r(w) \in R(W) \) and make all neighbors of \( r(w) \) all colored \( G \) and \( W \). Then by coloring \( c(r(w)) = B \) we block the white odd-ring \( R(W) \). Note that no Kempe’s tangling has occurred here since \( R(W) \) divides the dichromatic map \( M(B,G) \) into two parts and Kempe switching applied one at a time.

The map coloring algorithm has been illustrated on the Haken-Appel’s map shown in Figures 4-6. In Figure 6 although there is no white odd-ring we have demonstrated by the use of the Kempe-chains how the regions labeled by \( X \) and \( X' \) can be recolored by brown by freeing up all neighbor regions from the color brown. Binary labels shown in the figures shows parity of the sides of each regions of the islands and since \((mod 2)\) sums are all 0, each islands are surrounded by even white-ring.

**Theorem 2.** The map \( M(B,G) \) obtained by the Map-Coloring-Algorithm in Step 2 can be extended to a four coloring of \( M \).

**Proof.** Proof follows from Lemmas 2,3,4 and 6.

**Step 3.** Four coloring of \( M(B,G,1B, dB) \).

Since maximal dichromatic map \( M(B,G) \) has only even white-rings and acyclic white regions, i.e., forest of disjoint trees and paths we can easily color them with light-blue \( 1B \) and dark-blue \( dB \).

That is at the end of Step 3 the initial all-white normal map \( M \) transformed into four colored map of \( M(B,G,1B, dB) \) with the regions of high-lands, low-lands, deep-seas and shallow-seas.

From Theorem 2 we re-state the famous four color map theorem.

**Theorem 3.** All cubic planar maps are 4-colorable.
CONCLUDING REMARKS

We extract the following from the first page of Appel and Haken’s paper [3]:

The first published attempt to prove the Four Color Theorem was made by A.B. Kempe in 1879. Kempe proved that the problem can be restricted to the consideration of “normal planar maps” in which all faces are simply connected polygons, precisely three of which meet at each vertex. For such maps he derived from Euler’s formula the equation

\[ 4p_2 + 3p_3 + 2p_4 + p_5 = \sum_{k=7}^{k_{\text{max}}}(k - 6)p_k + 12 \]

where \( p_i \) is the number of polygons with precisely \( i \) neighbors and \( k_{\text{max}} \) is largest value of \( i \) which occurs in the map. This equation immediately implies that every maximal planar map contains polygons with fewer than six neighbors. In order to prove the Four Color Theorem by induction on the number \( p \) of polygons in the map \( (p = \sum p_i) \), Kempe assumed that every normal map with \( p \leq r \) is four colorable and considered a normal planar map \( M_{r+1} \) with \( r + 1 \) polygons. He distinguished the four cases that \( M_{r+1} \) contained a polygon \( P_2 \) with two neighbors, or a triangle \( P_3 \) or a quadrilateral \( P_4 \), or a pentagon; at least one of these must apply by the equation.

This beautiful Victorian Age deduction works for \( P_i, i = 2, 3, 4 \) and unfortunately fails for \( i = 5 \). I think no mathematician of that period would be able to guess the possible length of a proof in future based on reducibility.

In this paper, by choosing direct proof, that is the opposite direction of the above, we have given an algorithmic proof for the Four Color Theorem which is based on an coloring algorithm and avoiding three-colorability in a maximal two-colorable map. The last word about the proofs given in [6],[7],[8] and including this one that uses spiral chains in the coloring algorithm. Simply enable an efficient coloring algorithm and protect us to fall in a situation similar to Kempe-tangling.

Again Appel and Haken argue strongly that [2],[3]:

…it is very unlikely that one could use their proof technique without the very important aid of a computer to show that a large number of large configurations are reducible. Of course, this does not rule out the possibility of some bright young person devising a completely new technique that would give a relatively short proof of the theorem.

This paper does not prove the truth of the first sentence but it does prove that the second sentence is wrong, not only just because of the length of the proof.
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