Love Letter

It can’t go on and on like this!
This what we have we have to keep.
When we’ll meet,
Something must be.
Together singularity.

What’s up on singularity?
For sure no platitudes.
Because what’s singularity,
Is not wearing down so easily.
On a park bench in Grunewald
In two to face the rain,
Attempts in vain. Girl, take heart!

Write me a card! Yours, Bertie! (Erinner SIngularity).
Poem by Joachim Ringelnatz

Notes from algebraic geometry: Vertices or singularities are very interesting from the mathematical point of view. These points behave extraordinarily. Tiny changes in the equation have strong effects on these points.

Sweet \((x^2+9/4y^2+z^2-1)^3-x^2z^3-9/80y^2z^3=0\)
This surface described by the equation $x^2 + y^2 + z^3 = z^2$ was one of the very first visualizations we tried. Equation and shape are simple: A vertical alpha-loop rotates around the z-axis. But there was the problem with the colouring. Green is generally rather tricky in three-dimensional visualization of surfaces and, in addition, tends to be matt or yellowish. The lights and reflexions must be well tested. Note the light blue hard shadow intensifying the spatial effect.
The horizontal section through Cone is a so-called rosette curve: A small wheel rolls round the interior of an annular body while a pencil fixed to the wheel is drawing a curve. Different curves are formed depending on the ratio of the two radii. The curve closes if the ratio is a rational number. In our case it is the four-leaf clover. Our cone has the advantage that four scoops fit in. You must hurry licking otherwise the ice cream is dripping down.
This surface is, in fact, rather complicated, even if it features a regular mirror symmetry. Its name is due to its inward-turned identity: discrete, quiet and graceful. The loop-shaped bow makes one think of a kimono bow. The colours and the image result from ray tracing: A ray is traced from each point in the image and where it hits an object the colour value is calculated by means of light source models. The grace of Geisha is underlined by special lighting.
This surface was developed by coincidence on a drab train ride (Working with algebraic visualization makes time elapse quite fast). But the constellation isn’t a coincidence at all, on the contrary, if one wanted to systematically derive the algebraic equation for this surface one would face unsurmountable problems. The challenge of Miau is, of course, the double opening with embedded singularity. For mathematicians this is a treasure trove exploring the relationship between equation and form.

Miau \[ x^2yz+x^2z^2+2y^3z+3y^3=0 \]
A piece of paper is folded and is held from beneath such that you can put your four fingers in the four corners so formed. By spreading your fingers the figure opens in two different ways so that two of the four inner sides can be seen at a time, the blue ones for heaven, the red ones for hell. Children guess which colour will show up. Our figure reminds us of this game, hence the name. By adding up the squares at $y$ and $z$ you get the highest exponent 4. This is called an equation of the 4th degree. The higher the degree the more complicated it is to calculate the surface.
The upper border of this surface is a loop in the shape of the Greek letter Alpha, whereas the border to the right consists of two parallel so-called cuspidal curves with a spike each. By moving the horizontal loop downwards, leading its two endpoints on the right along the curve, the surface Quaste will come out. On the other hand, you can also shift one of the curves along the loop and you will get the same figure. Surfaces with this property are named Cartesian products after the French mathematician René Descartes.

\[8z^8 - 24x^2z^6 - 24y^2z^6 + 36z^8 + 24x^4z^4 - 168x^2y^2z^3 + 24y^4z^2 - 72x^2z^5 - 72y^2z^5 + 54z^7 - 8x^6 - 24x^4y^2 - 24x^2y^4 - 8y^6 + 36x^4z^2 - 252x^2y^2z^2 + 36y^4z^2 - 54x^2z^4 - 108y^2z^4 + 27z^6 - 108x^2y^2z + 54y^4z - 54y^2z^3 + 27y^4 = 0\]
If you want to find the equation of this surface it would take strong efforts. The soft tangential contact is not easy to achieve. It vanishes as you only slightly change the formula.
The elegance of the sea horse is an illusion: If you look at it from behind or from the side, it appears quite clumsy. Sea horses live worldwide in tropical and temperate climate zones. Its Latin name is Hippocampus, you can find the equation next to it.

Seahorse $(x^2 - y^3)^2 = (x + y^2)z^3$
Vis à Vis means opposite – and, here, two essential phenomena of algebraic geometry stand opposite each other. The singular tip on the left looks at a curved but smooth hill on the right. This singularity is more exciting, because various changes to the equation can result in unpredictable changes to the figure, which does not happen at smooth points. By using the SURFER program, which is available for free at www.imaginary.org such surfaces can be generated and modified quite easily and intuitively. The comparison of form and formula, i.e. of equation and corresponding surface, becomes an interactive experience which is intriguing to understand.

Vis à Vis $x^2-x^3+y^2+y^4+z^3-z^4=0$
Though an algebraic surface is called “Sofa” sitting on it is not necessarily comfortable. What we have here, rather, is a seating for two separated from each other by a singularity. This singularity is called E8 in mathematical language and is perhaps the most famous of all singularities. It combines, among others, the theory of the symmetry groups of Platonic solids (E8 is part of the icosahedral symmetry group) and the theory of the Lie groups. The real image of this singularity excels through its elegance, though it does not reveal its mathematical complexity, which only becomes apparent as you include the imaginary part.
The equation \( x^2 + z^2 = y^3(1-y)^3 \) of Citric appears as simple as the figure itself. Two cusps mirror-symmetrically arranged rotate around the traversing axis. The equation \( x^2 + z^2 = y^3 \) simplified by omitting \((1-y)^3\) provides for exactly one cusp, and \( x^2 + z^2 = (1-y)^3 \) yields the mirror image. Both are infinitely extending surfaces. The product on the right side of the initial equation ensures that Citric remains bounded. You may consider the following: If the absolute value of \( y \) is getting larger than 1 the right side becomes negative and the equation does not admit real solutions of \( x \) and \( z \).

**IMAGINARY**

The equation \( x^2 + z^2 = y^3(1-y)^3 \) of Citric appears as simple as the figure itself. Two cusps mirror-symmetrically arranged rotate around the traversing axis. The equation \( x^2 + z^2 = y^3 \) simplified by omitting \((1-y)^3\) provides for exactly one cusp, and \( x^2 + z^2 = (1-y)^3 \) yields the mirror image. Both are infinitely extending surfaces. The product on the right side of the initial equation ensures that Citric remains bounded. You may consider the following: If the absolute value of \( y \) is getting larger than 1 the right side becomes negative and the equation does not admit real solutions of \( x \) and \( z \).

**IMAGINARY** is an open platform for mathematics exhibitions by the Mathematischen Forschungsinstituts Oberwolfach.

If the audience in an oval stadium scream about a score (typically of the favoured team), the sound spreads like a quickly inflated floating tyre. After some split seconds the tyre meets itself at its centre – the opening has closed – and that is what exactly happens at the kick-off spot. At that point sound waves from all directions meet simultaneously and are boosted accordingly. This is why referees are advised to always stay level with the ball. Thus, when a goal is scored they do not stand in the middle circle and get a buzzing in their ears.

$Dullo \quad (x^2+y^2+z^2)^2-(x^2+y^2)=0$