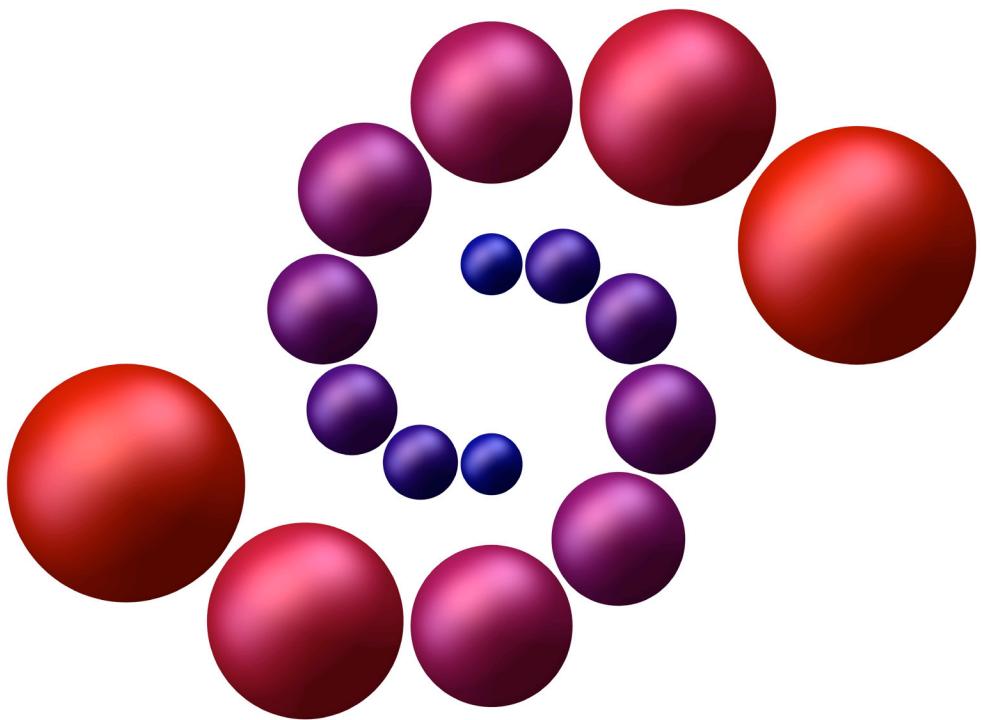


# NIMS IMAGINARY

## EXHIBITION GUIDE



Spiral Spheres

$$0 = \prod_{k=0}^1 \prod_{l=0}^7 \left( \left( x - i^{2k} \psi_x \left( \frac{\pi}{5} l \right) \right)^2 + \left( y - i^{2k} \psi_y \left( \frac{\pi}{5} l \right) \right)^2 + z^2 - \left( \frac{\pi}{10} \|\Psi\left(\frac{\pi}{5} l\right)\| \right)^2 \right)$$

$i^2 = -1, \quad \Psi(t) = (\psi_x(t), \psi_y(t)) = (e^{bt} \sin t, e^{bt} \cos t)$   
 $b = \frac{2 \ln \phi}{\pi}, \quad \phi = \frac{1+\sqrt{5}}{2}$  (golden ratio)

Edited by  
National Institute for Mathematical Sciences (NIMS)  
Mathematisches Forschungsinstitut Oberwolfach (MFO)

[www.nims.re.kr](http://www.nims.re.kr)  
[www.mfo.de](http://www.mfo.de)  
[www.imaginary.org/nims](http://www.imaginary.org/nims)

IMAGINARY is supported by the Klaus Tschira Stiftung.  
Title image: Spiral Spheres by Bianca Violet

The book is available under  
the Creative Commons License CC-BY-SA-NC-3.0 and  
can be downloaded at [www.imaginary.org/nims](http://www.imaginary.org/nims)

Seoul, August 2014

NIMS  
IMAGINARY

## EXHIBITION GUIDE

# Welcome to the NIMS-IMAGINARY exhibition

It is so exciting to invite you to this NIMS-IMAGINARY exhibition!

21st century mathematics is not only at the frontier of human understanding of the universe, but also changing the landscape of technology leading to unforeseen industrial development. Mathematics popularization raises awareness of such expanding roles of mathematics in 21st century. In this era of rapid scientific development and discoveries, it is becoming ever more important for scientific researchers to interact with the general public. IMAGINARY is one such attempt bridging mathematics and the public.

By joining the IMAGINARY team, you will touch and see mathematics!

Hyungju Park  
Chair NIMS-IMAGINARY

NIMS-IMAGINARY is a project by the National Institute for Mathematical Sciences (NIMS) in collaboration with the Mathematisches Forschungsinstitut Oberwolfach. IMAGINARY is supported by the Klaus Tschira Stiftung.

6	Introduction
8	SURFER
12	Morenamenti
14	Cinderella
18	Crystal Flight
18	FroZenLight
20	jReality
22	Qi
24	Dune Ash
26	TsunaMath
28	The Future of Glaciers
30	The Sphere of the Earth
34	3D Printing
36	How to organize an IMAGINARY exhibition?
40	The National Institute for Mathematical Sciences
42	The Mathematisches Forschungsinstitut Oberwolfach
44	Credits
	5

# NIMS-IMAGINARY – interactive and open mathematics in South Korea

IMAGINARY started in 2008 as an interactive travelling exhibition that presented visualizations, interactive installations, virtual worlds, 3D objects, and their mathematical background in an attractive and understandable way. Over the last years it has developed into an open source organization including a web platform ([www.imaginary.org](http://www.imaginary.org)) and a network of people motivated by interactive and participative math communication.

The aim of the platform »IMAGINARY – open mathematics« is to provide a space for the presentation and development of math exhibitions. All contents of IMAGINARY are made available to a broad audience under a free licence and can thus be reproduced and used for individual exhibitions and events. Moreover, the platform provides an opportunity for everyone interested to contribute with their own material and serves as a hub for the exchange of ideas in the field of math communication, a field that has been very dynamic in the last years. The target group of the platform includes museums, universities and schools.

The platform currently hosts two exhibitions: the original exhibition »IMAGINARY – through the eyes of mathematics« that was the starting point for the whole project, and a second exhibition called »Mathematics of Planet Earth« which was organized for the worldwide initiative of the same name.

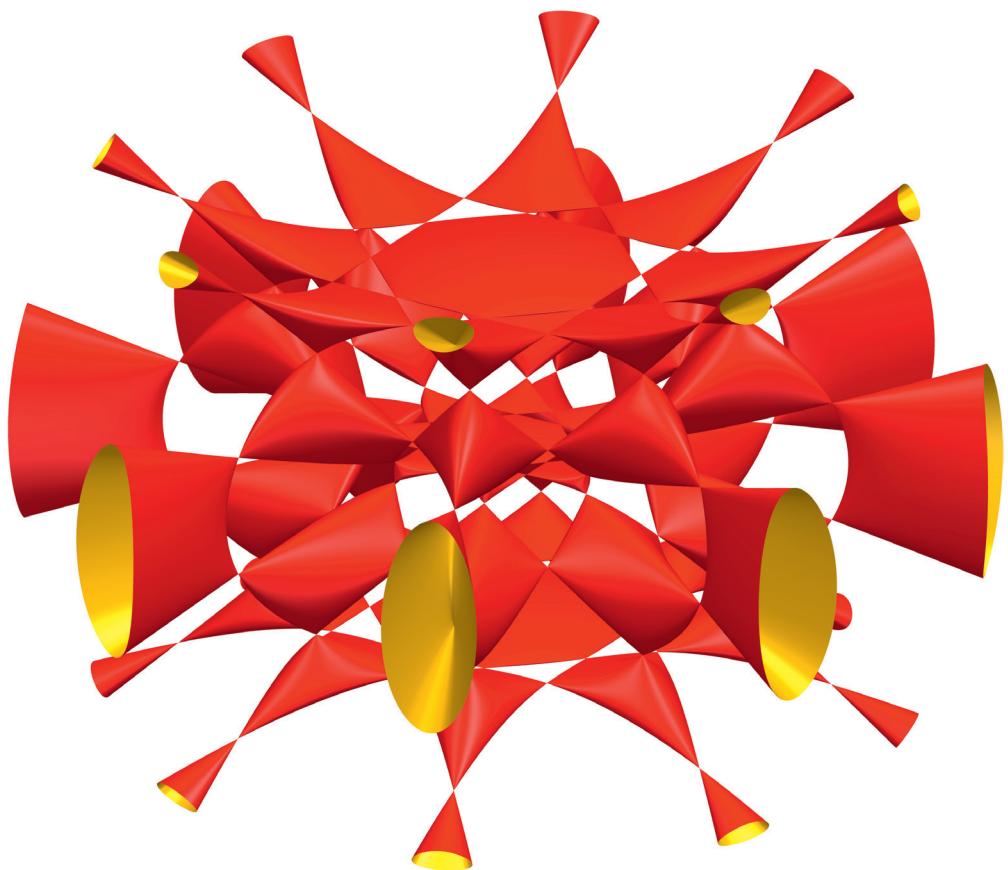
»IMAGINARY – through the eyes of mathematics« was designed by the Mathematisches Forschungsinstitut Oberwolfach (MFO) and was supported from 2008 to 2009 by the German Federal Ministry for Education and Research (BMBF).

From 2011 to 2016 IMAGINARY is supported by the Klaus Tschira Stiftung. The exhibition has visited more than 125 cities in over 30 countries and there are even some permanent installations, for example in the Deutsches Museum in Munich and in the National Museum of Mathematics in New York. The didactical and aesthetic concept aims to create enthusiasm for mathematics and sets out to spur interest and curiosity for the theoretical background, with the help of visual impressions, live-interactions, background materials available on the web platform, as well as guided tours and individual assistance on site.

### NIMS-IMAGINARY

The National Institute for Mathematical Sciences (NIMS) has decided to bring IMAGINARY to South Korea and to stage a unique exhibition during the ICM 2014 in Seoul. For this, all IMAGINARY contents have been translated into Korean and many programs and exhibits have been updated and revised. Besides the exhibition in Seoul, several other activities will be carried out. NIMS-IMAGINARY will be permanently installed at NIMS in Daejeon and a travelling exhibition is planned for the future. Korean mathematicians, school teachers and the general public will be invited to participate and further develop the contents.

You can find more information on NIMS-IMAGINARY and all programs to be downloaded under an open source license at:  
[www.imaginary.org/nims](http://www.imaginary.org/nims)





SURFER is a program for the visualization of real algebraic geometry in real-time. The surfaces shown are given by the zero set of a polynomial equation in three variables. The program was first developed for the exhibition »IMAGINARY – through the eyes of mathematics«.

SURFER allows the user to enter a polynomial equation and shows the corresponding surface in space as a beautiful image. The use of the program is very easy and intuitive. The created surfaces can be rotated and recolored on the screen and there is also a picture gallery with explanations for interesting surfaces with special characteristics.

## How are formula and form related to each other?

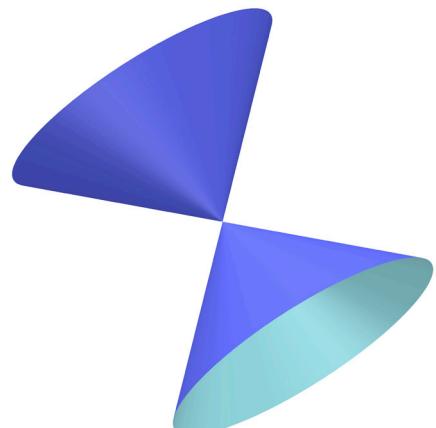
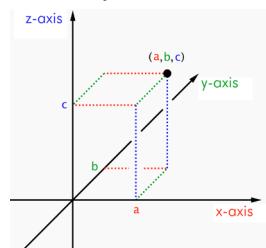
Algebraic geometry deals with the connection between algebra and geometry, here for example a formula (algebra) and the form (geometry) it describes.

The physical space in which we move, the space that surrounds us, is three dimensional. By placing a set of three coordinate axes perpendicular to each other, we define an origin as well as the three directions x, y and z. This enables us to orient ourselves by referring to them.

They define the three coordinates x, y and z of a location P. In turn, these are the three numbers which set and describe the position of a point after a successful selection of our coordinate system.

In SURFER, equations entered use the three variables x, y and z corresponding to these three axes.

Let's take  $x^2+y^2-z^2=0$ . For a point to be located on the appearing surface means that its coordinates satisfy the equation. The surface is therefore constituted of the set of all the triples  $(x,y,z)$  which yield zero when they are substituted into the equation. (You could say  $(x,y,z)$  is a 'zero' of the equation).



For example: The point  $(x,y,z)=(1,0,1)$  satisfies the equation  $x^2+y^2-z^2=0$  because  $1^2+0^2-1^2=0$ . Therefore  $(1,0,1)$  is a point on the corresponding surface.

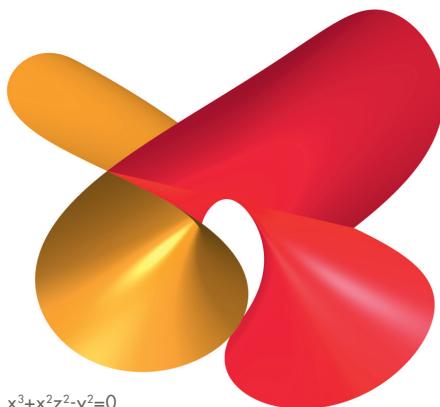


## References

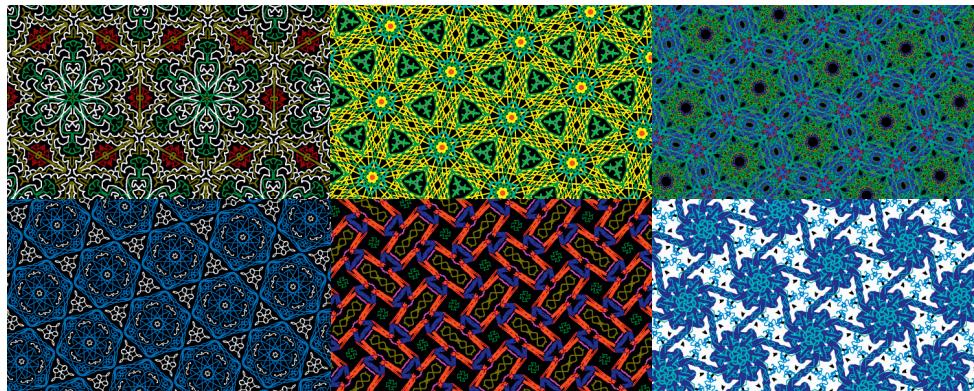
- SURFER manual and SURFER expert tricks on [www.imaginary.org/program/surfer](http://www.imaginary.org/program/surfer)
- Articles: »Curves and cusps« by Duco van Straten, »Resolution of singularities« by Herwig Hauser and »World record surfaces« by Oliver Labs in the texts/background material section of the IMAGINARY platform
- Book: »Undergraduate Algebraic Geometry« by Miles Reid, Cambridge University Press, 1988

## Activities

- *Your own surface:* create your own algebraic surface, print it using the print button, and take it home!
- *Image galleries:* Explore the three image galleries in the SURFER program and read the explanation texts for each of the surfaces shown. The »Tutorial« gallery gives you a basic introduction into SURFER and algebraic geometry, and features mathematical tricks how to create beautiful surfaces. The »Fantasy Surfaces« gallery presents several artistic surfaces. Try to change the heart equation (surface »Sweet«). What happens if you replace the last cube by a square? In the »World Record Surfaces« gallery you will find the most singular surfaces. How many singularities can a surface of a certain degree have?



# Morenements



Patterns created with Morenements

With the program Morenements you can tessellate a plane based on a basic pattern, meaning you can fill the whole plane with a pattern (ornament) without gaps or overlaps. Just draw on the black space with your finger, select color and thickness of your virtual pen on the right hand side and watch as the basic pattern on the upper right side fills the plane.

Interestingly enough, all possible patterns can be traced back to one of the 17 so-called symmetry groups that can all be set on the left side of the screen and are mathematically represented by simple letter abbreviations (e.g. p1, p2, pm, pg, etc.).



A »tile« is a part of the plane that is composed of mirrored and rotated versions of the basic pattern. An »ornament« extending indefinitely is created by copying this single tile and placing the copies next to each other. Astoundingly enough, only a finite number of structural possibilities exist.

That means there are only a finite number of different specifications of how a tessellation can be created by mirroring, rotating and shifting of a set basic pattern. There are exactly 17 and they are referred to as the 17 »plane crystallographic groups« (or »wallpaper groups«, or »plane symmetry groups«).

The famous mathematician Hermann Weyl defined symmetry like this: »A structure is symmetric when it can be changed in some way and the result is the same thing as in the beginning«. From this point of view the tessellations obtained in Morenements are highly symmetric structures. Each tessellation can be perfectly converted to itself through mirroring, rotation and shifting. The set of all operations that convert the tessellation to itself is called a "symmetry group" in mathematics.

These 17 symmetry groups were already known to the Arabs during the Nasrid period, the time between 1232 and 1432. You can find them in the Arabic ornaments all through the Alhambra, a fortress in the Spanish city of Granada.



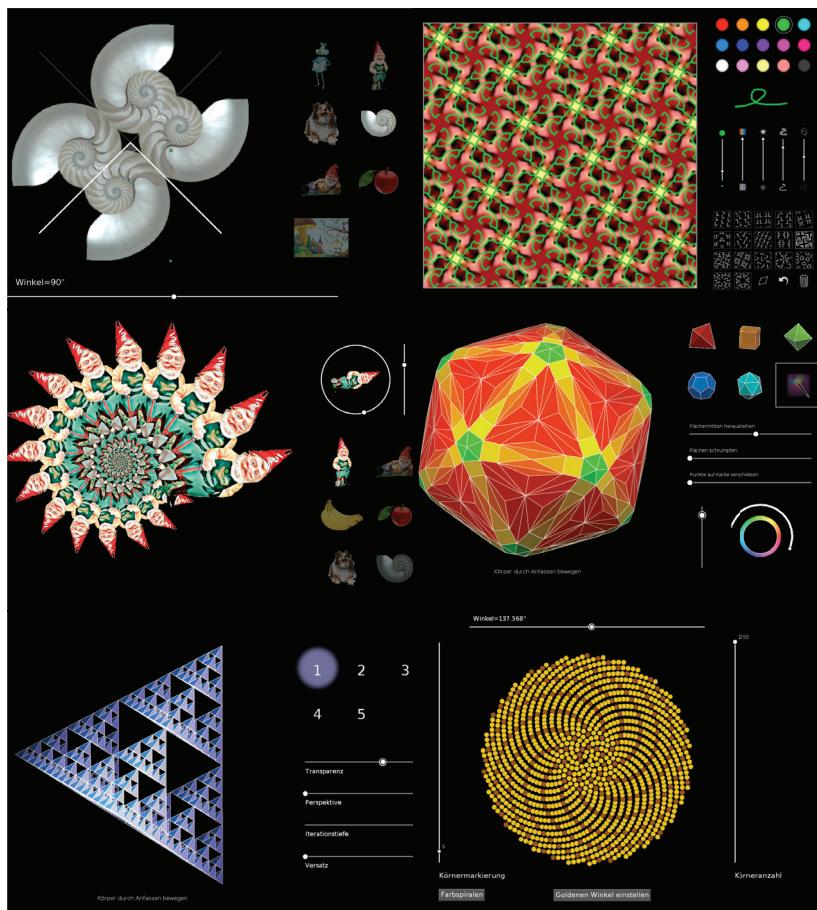
Ornament of the Alhambra wall decoration

## References

- ☰ Information on the 17 symmetry groups can be found in the Wikipedia article on wallpaper groups.
- ☰ Within the program Cinderella, there are a couple of experiments with symmetry. There is also one where you can paint ornaments with additional settings (brush size and opacity) for creating more artistic images.
- ☰ For iPhone/iPad you can download the »iOrnament« program from the app store, which offers additional descriptions and even more drawing options. See also [www.science-to-touch.com/iOrnament](http://www.science-to-touch.com/iOrnament)

## Activity

- ⌚ Your own ornament: paint your own ornament, print it using the print button, and take it home!



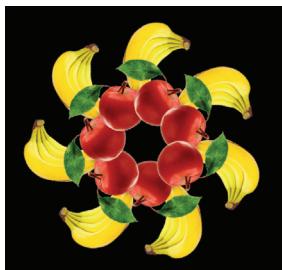
Six different Cinderella-experiments from the subject areas »plane mirroring«, »rotation and compression«, »platonic bodies«, »spatial package« and »Mathematics and plants«.



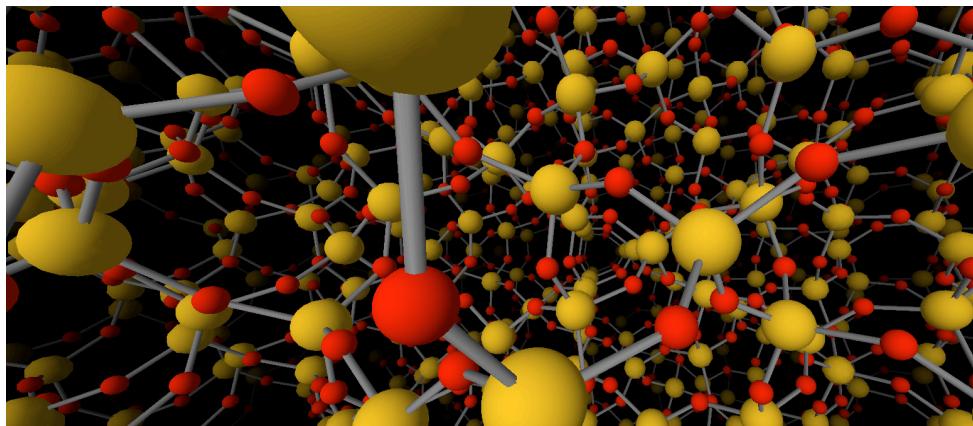
This collection of experiments delivers insight into many mathematical subjects, such as symmetry, chaos and simple physical simulations. The experiments are interactive, which means for every example there is the chance to change parameter, object and characteristics, and to observe the consequences of these changes in real-time.

You can play with a robot that has to descend and ascend a staircase, experiment with the orbit of planets revolving around one or several suns, watch shoals of fish and change the size of the swarm, explore symmetric objects, let sunflowers grow and many things more.

All these experiments were developed with the program Cinderella, a dynamic geometry software.



# Crystal Flight



Virtual flight through a Quartz crystal

Crystal Flight is an interactive program that takes you on a journey through the inside of a quartz, fluorite or diamond crystal. Flying a miniature spaceship you can explore the crystal structures even in 3D!

Matter exists mostly in three kinds of states or phases: gas, liquid and solid matter. Liquid and gaseous matter is often disorganized or amorphous and chaotic, but solid matter can form beautifully arranged structures. In many minerals the atoms are organized in highly ordered crystal lattice structures.

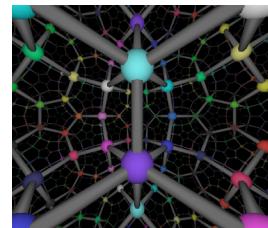
The three structures displayed here are representatives of three classes of crystal structures that form different geometric patterns each. In a diamond each atom is surrounded by four of the same atoms in the form of a triangular pyramid. This is called the diamond cubic crystal structure and there are many different materials that adopt the same structure as they solidify, such as silicon/germanium alloys or tin.



Fluorite minerals are made from at least two different atoms. The most common combination is calcium fluorite, where each calcium atom is surrounded by eight fluorine atoms in a cubic box shape and each fluorine is surrounded by four calcium atoms in the form of a triangular pyramid. Other minerals adapt this cubic structure as well, for example lithium oxide.

The quartz structure is the most complex of the three. The mineral contains silicon and oxygen in a one to two ratio and forms triangular pyramids with silicon at the centre and oxygen at the corners. These pyramids can be arranged in different ways depending on the type of quartz and temperature.

There are also two other visualizations which look similar to crystal structures but which are curved abstract mathematical spaces. The first (colored) visualization allows you to fly through the famous dodecahedral space of the French mathematician Henry Poincaré (1854-1912). This space is formed by identifying each face of a dodecahedron with its opposite face, using the minimal clockwise twist to line up the faces. The other (3D) space is the hyperdodecahedron of the Swiss mathematician Ludwig Schläfli (1814-1895). Here, 120 dodecahedra are glued together to form a hypersphere.



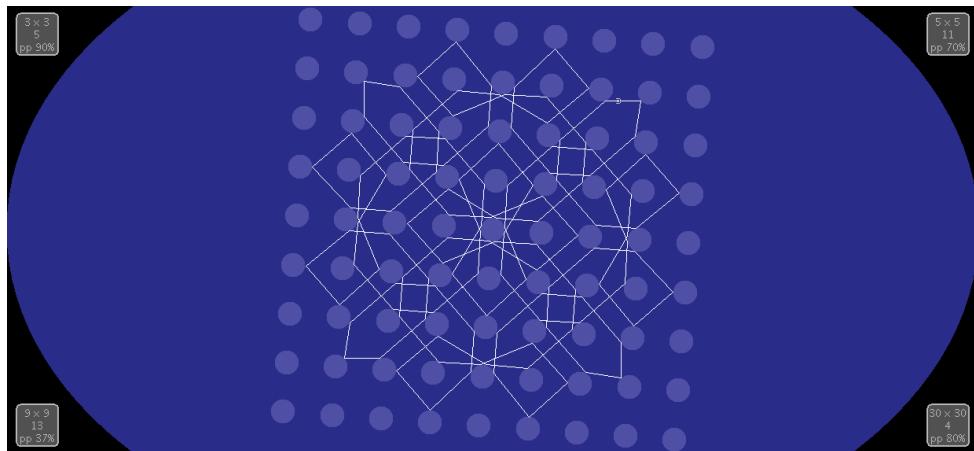
Mathematical space flight:  
Hyperdodecahedron

With Crystal Flight you can explore these structures closely. Steer your flight with your finger over the touch screen and regulate the speed with the controller at the bottom right corner.

#### Activity

- ⌚ **3D flight:** to enjoy your flight in 3D, simply put on the 3D anaglyph glasses provided next to the exhibit.

# Frozen Light



Reflecting light pattern in a 9x9 mirror grid

The program FroZenLight simulates a perfect ray of light reflecting on circular mirrors. As simple as it sounds, one can explore optics and geometry, but surprisingly this exhibit also gives an insight into chaos, art and cryptography.

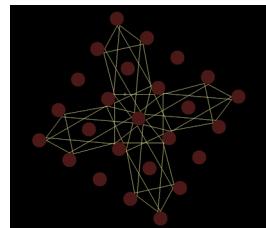
The law of reflection on a mirror surface states that the incident ray and the reflected ray form equal angles with the normal direction (that is, with the radial direction for a circular mirror). The program considers circular mirrors arranged in a grid-like manner, that reflect a perfectly straight light ray that never diminishes.

After a few initial tries, it becomes clear that it is quite difficult to predict the direction of the ray after a couple of reflections. Random positions of the light source produce chaotic reflection patterns. However, it is possible to position the light source so that symmetric reflection patterns are created, resulting in beautiful and artistic images.



Click on one of the sample symmetric patterns from the patterns menu. You can change the position of the light source and rotate the grid of mirrors. These actions will destroy the delicate pattern, and you won't be able to recover it. The precision required for recovering the pattern is just too high to be achieved by manual adjustments.

The program looks for symmetric patterns. Each button contains the essential information that generates the pattern. The first data is the grid size ( $3 \times 3$ ,  $5 \times 5$ , ...). The second is a number that constrains the number of reflections of the minimal path that generates the pattern by symmetry. To reveal the generating path, click on the button with a question mark (?). You will see that the minimal generating path touches the announced number of circles. The »p« and »g« symbols encode the position of the endpoints of the generating path, and the percentage encodes the size of the circles. There are three more special buttons to change the size of the circles, to browse through various color patterns, and to start an automatic animation that will look for symmetric patterns by continuously moving the grid and light source.

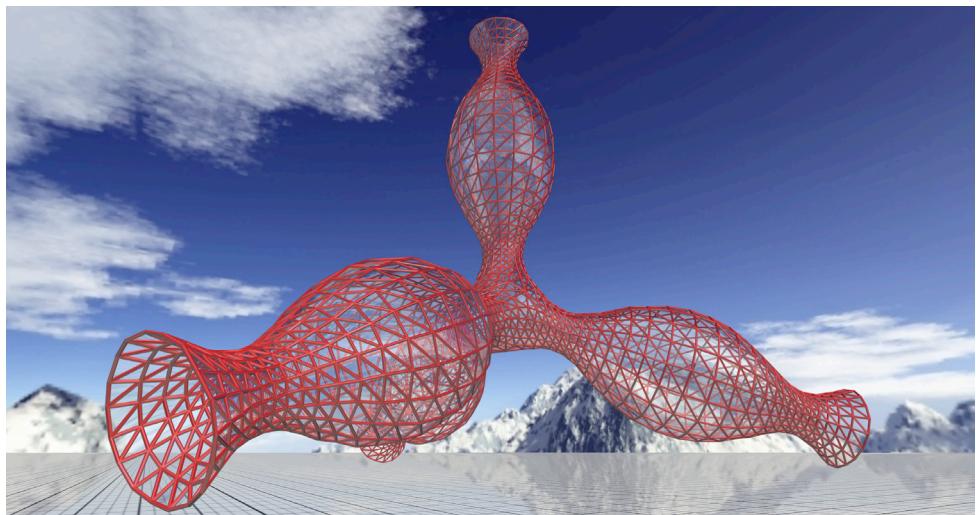


Reflecting light pattern in a  $5 \times 5$  mirror grid

The data shown in the buttons are constraints for the problem of finding such a light pattern. One can actually prescribe an arbitrary sequence of reflections in a grid, and create »pixelated« patterns such as the example of 1807 reflections. With four adjacent mirrors, one can prescribe a sequence for each letter in the alphabet, so each square of four mirrors in a grid provides a slot for a character in a message. This constitutes the basics of a cryptographic system. The example with 3517 reflections represents an encrypted text. The encoded text is given just by the height of the light source and the rotation of the grid. At least one of these numbers will require a high floating point accuracy, meaning a large number of decimal digits, to be able to store all the information of the original message.

## References

- ≡ In the full version of FroZenLight (to be downloaded) you can experiment with the extended encryption options
- ≡ Try the »mirror circle« applet in the exhibit Cinderella. With it, you can place the light source also inside the circle.



The tetratoid

This program is a 3D simulator of a virtual environment, such as a first-person video game. You can walk, fly and shoot balls at virtual sculptures related to shapes from differential geometry.

Differential geometry studies the bending of surfaces and other objects in space, the measure of distances and angles in these surfaces, and other precise features such as the curvature of a shape.

The sculpture of the Matheon bear is a recreation of a sculpture at the technical university of Berlin. Although this is not a mathematical shape, an accurate description of the surface allows to draw the painting on it. The bear is painted with circles and logos of the university. The circles are not really circles, since they are not on a flat surface. However, the angles of the hexagonal grid and all angles on the flat design are preserved on the curved surface of the bear. The image of the pattern is said to be conformal to the original one.



The Boy surface is a realization of an abstract object called projective plane. It is obtained by gluing a Möbius strip and a disc along their boundary. If it is difficult to imagine as abstract rubber surfaces, it is even more challenging with the Boy surface realization. This surface has the property of having the lowest possible mean curvature, which is to say that it is the least bended surface that realizes the projective plane.

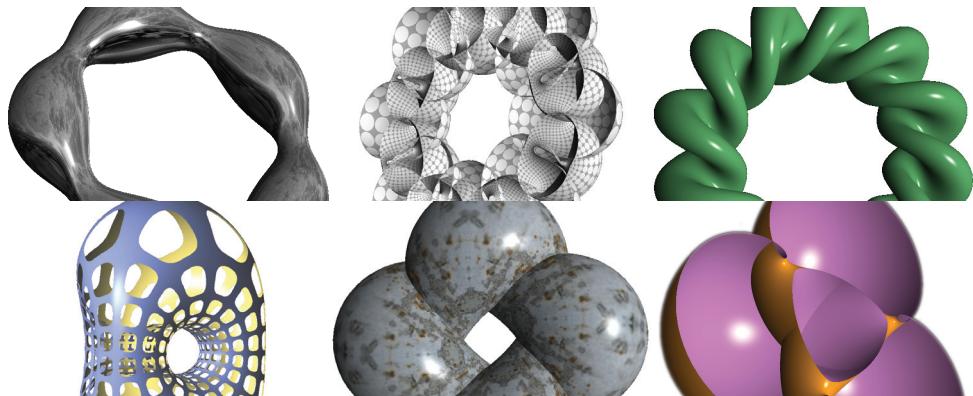
Minimal surfaces are those that locally minimize the area, saving material. This phenomenon appears physically in soap films, that reduce the exposed area due to the surface tension on the water and soap layer. These tensions are balanced in a way that can be interpreted in terms of the mean curvature, which must be zero. The Enneper surface shown in the program is one such example, although it would be almost impossible to recreate with soap film. The surface actually extends further towards infinity, and it soon self-intersects. This kind of properties are global, in contrast with the local information that gives the mean curvature restriction.

The Schwarz minimal surface is another example of a minimal surface, this time a periodic structure that fills the space with a cubic-like grid. However, the representation here in jReality is not a curved surface, but a collection of flat discs contacting each other and making a »discrete« approximation of the surface. This requires a mathematical theory to approximate curved surfaces with flat elements, which is a useful tool in architecture and in computer graphic design.

Soap films consist of a material which resists being stretched. However, you can bend it in any direction with no effort. Willmore surfaces, in contrast, consist of a material which does not resist stretches, but develops elastic counter force against bending. The Willmore torus is one such example.

The Tетranoid is a close relative to minimal surfaces. Whereas minimal surfaces have zero mean curvature, the tetranoid has non-vanishing constant mean curvature.

Helicoids are surfaces resembling a snail staircase. These are minimal surfaces. One can modify the surface by connecting two levels of the staircase with a hole. The surprising property is that this operation can be done while the surface stays minimal, thus obtaining a Helicoid with handles.



Surfaces from Qi. From top left: Delauney, Dobrinier, Willmore, Lawson, Twisted De launy and Wente

Qi visualizes surfaces which arise as equilibria shapes under certain »energy« constraints. Surfaces of constant mean curvature (CMC) minimize surface area enclosing a fixed volume. Willmore surfaces minimize the bending energy of the surface.

Mathematically, these »energy« constraints can be expressed by non-linear partial differential equations and the surface is a solution to these equations. Ideally, one would like to describe the structure of the space of all solutions (moduli space). Over the past 30 years methods from geometric analysis (providing existence of solutions in a general setting) and integrable systems theory (providing all solutions in more specialized situations) have contributed substantially to our understanding of this problem.



The surfaces shown in Qi have all been computed using methods from integrable systems theory which allows for a description in terms of holomorphic data. This description is especially amendable to numerical computation and experimentation. In current research these experiments are used to unveil the structure of the moduli space. Further, they help to form and find evidence for conjectures, which later on motivate rigorous proofs.

*Delaunay:* This embedded torus of revolution has constant mean curvature in the 3-sphere. It can be constructed by rotating an elastic curve in the Poincaré disk around the boundary at infinity. Recent results by Simon Brendle (2012), Ben Andrews and Haizhong Li (2012) show that all embedded CMC tori are obtained this way..

*Twisty Delaunay:* This torus is equivariant and non-rotational, with a 1-parameter group of isometric symmetries. It has constant mean curvature in the 3-sphere and was found by Martin Kilian, Martin Schmidt and Nicholas Schmitt (2010) via integrable systems methods.

*Wente:* The Wente torus, the first known example of an immersed torus in Euclidean space with constant mean curvature, was found by Henry Wente in 1985 via analytic methods. Earlier, Hopf had conjectured that the round sphere was the only compact and immersed constant mean curvature surface in Euclidean space. Uwe Abresch (1987) was the first to visualize this torus by computer.

*Dobriner:* The coordinates of this constant mean curvature torus were computed by Hermann Dobriner in his thesis in the 19th century. It is a counterexample to the Hopf conjecture. Before computer visualization, no one noticed the surface is actually compact.

*Willmore:* This torus, with a one-parameter group of isometric symmetries, is a critical point for the Willmore energy. It was found by Dirk Ferus and Franz Pedit (1990).

*Lawson:* The Lawson surface is the only known minimal surface of genus two. It was found by Blaine Lawson Jr. (1970) using the existence of a solution to a Plateau problem. It has various symmetries and is built out of a single piece by rotation and reflection. The grid lines on the surface show numerical conformal curvature lines. This parametrization was found in experiments by Nicholas Schmitt and Sebastian Heller (2013) using integrable systems methods.

## Dune Ash

On May 21st 2012 around 5:30 am the volcano Grimsvötn on Iceland erupted. The resulting ash cloud lead to the closure of airspaces across Scandinavia and Scotland and caused disruption of air travel throughout Europe.

The program Dune Ash carries out real time simulations of the dispersion of an ash cloud under the influence of given wind conditions, supplied by the user to the program. In this way it is possible to observe how the ash is distributed across Europe from the moment of eruption. The calculations are done in real time. This means it can take a while to complete even on powerful computers.



## How can the dispersion of ash clouds be simulated mathematically?



The transport process of single ash particles is modeled mathematically with a class of equations called partial differential equations, which are solved numerically by the program Dune Ash. After the program starts you choose the position of the volcano, and with that the eruption site. Using the arrow button the program guides you through the next steps. Next you sketch the wind conditions (wind field), by drawing lines on the screen. In this way the main wind direction is chosen. The wind velocity is derived from the speed at which you draw the lines across the screen. Based on the lines you drew, the wind field will be calculated and displayed across the map of Europe. The blue arrows signal the wind strength; a light color means lower velocities and a darker color higher velocities.

During the simulation the wind velocity is kept constant, i.e. it does not change with time as expected in reality. Additionally, the dispersion of the ash is regulated by so called diffusion. The influence of diffusion on the simulation is regulated by a slider. With the wind field and diffusion constant as input, the program calculates the solution of the partial differential equation in real time. The spatial and temporal dispersion of the ash will then be drawn onto the screen.

With the slider at the bottom right corner of the screen, it is possible to display the solution at different times onto the screen. With the »start/pause« button the animation can be halted and with the »stop« button the simulation is finished. Once the solution is calculated, the velocity field can be displayed at different resolutions as well as the grid that was used for solving the partial differential equation.



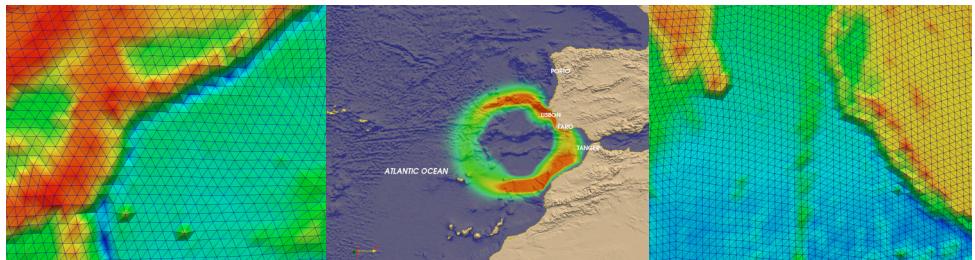
Volcano in Berlin with ash cloud over France

### References

- ☰ Dune Ash uses the modular programming package »Dune« for solving partial differential equations. ([www.dune-project.org](http://www.dune-project.org))

### Activities

- ⌚ *Ash cloud from Iceland:* Try to find a map displaying the ash cloud observed in 2011 and try to reconstruct it using the Dune Ash program. Do you observe differences between the model and reality?
- ⌚ Experiment with different wind fields and diffusion parameters. Under which condition does the ash cloud disappear after a given time and when does it accumulate in a given region on the map?



Simulation of tsunamis in Japan, Portugal and Indonesia

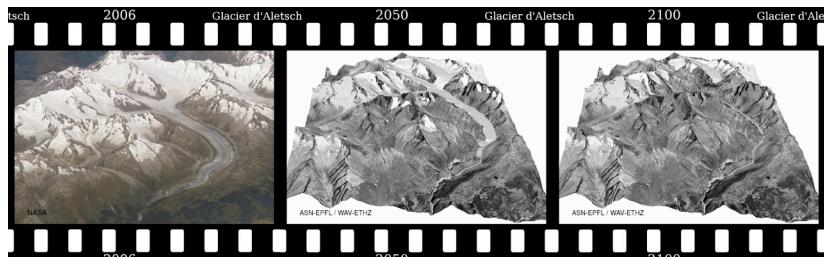
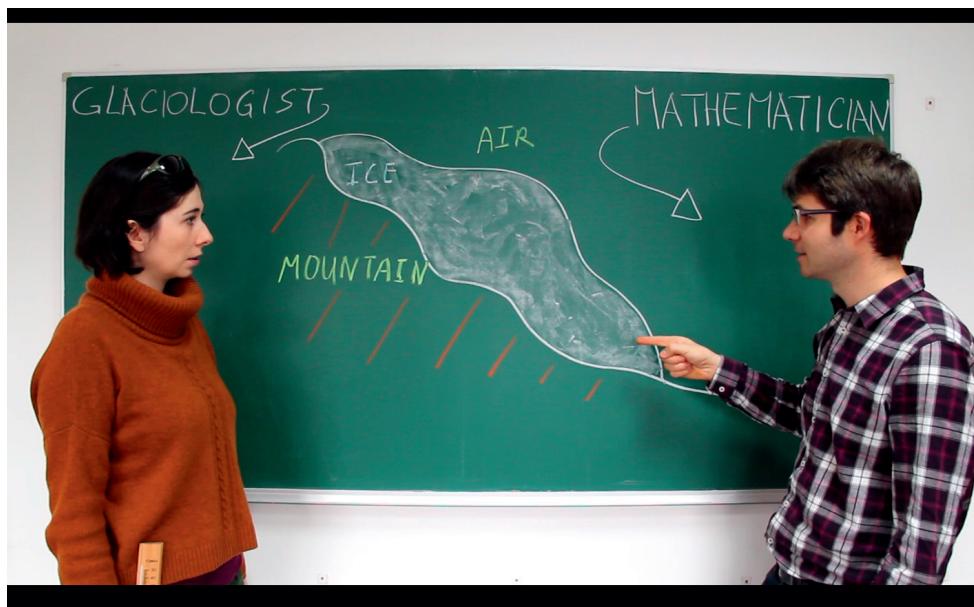
Tsunamis are big oceanic waves that collide violently with the coast. Most often, tsunamis are created by earthquakes that produce a sudden change on the topography of the ocean seabed. This exhibit explains how tsunamis are modeled mathematically, and recreates simulations of historical catastrophes.



The relationship between the height of the tsunami wave and the magnitude of the seism that causes it is complex. The speed, acceleration, displacement and size of the tectonic plates affect the height of the wave and its initial velocity. Once the wave is traveling across the ocean, the depth of the sea influences also the height and velocity of the water, and once the wave hits the coast, the slope and the topography of the seabed and coastline modify significantly the wave.

Mathematically, the variables to be calculated are the height of the wave (water depth), and the velocity of the water at each point of the map. The data to be known beforehand is the height of the seabed at each point (bathymetry) and the local modification of this seabed due to the seism. The so called Saint-Venant equations, or shallow water equations, are a set of partial differential equations that model the relations between these variables and data. These equations cannot be solved explicitly, but numerical algorithms can give accurate approximations that allow to obtain good simulations of the wave of a tsunami.

Using these methods, the exhibit shows simulations of great historical tsunamis, such as the Crete tsunami of 365 AD, the Lisbon tsunami of 1755, the Sumatra tsunami of 2004 or the Japan tsunami of 2011. These reconstructions are based on historical documents and descriptions or, in the more recent cases, can be checked against experimental observations to evaluate the validity of the model. Although earthquakes and tsunamis are difficult to predict, these simulations, together with measurements of active seismic zones, can help to reduce the potential impact of tsunamis on populated areas. Building realistic scenarios of affected areas can be used to plan infrastructures accordingly.



# The Future of Glaciers



How can one predict the evolution of glaciers?

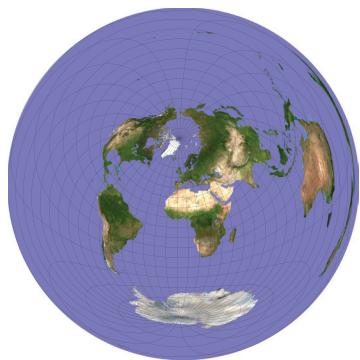
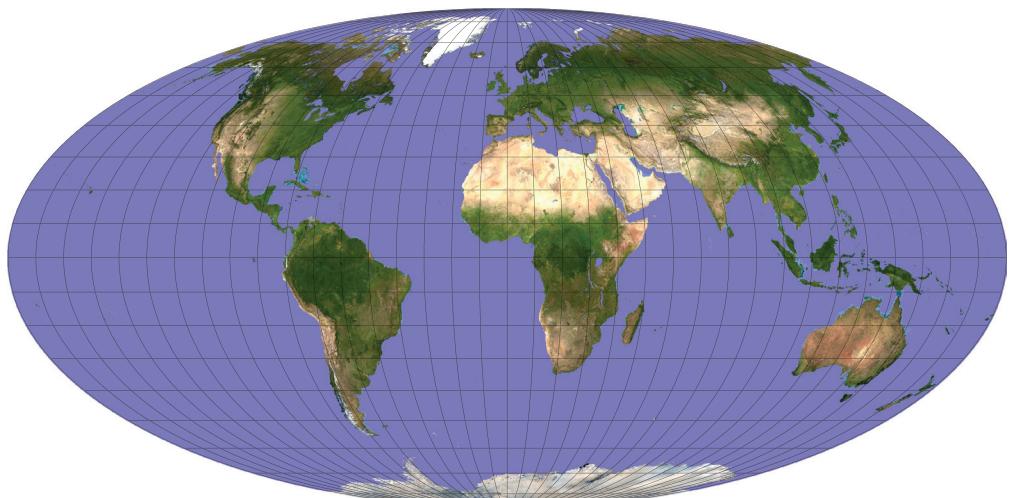
Alpine glaciers have been shrinking for more than a century. This trend is expected to continue if global warming progresses. This film shows how mathematicians and glaciologists work together to produce realistic estimates of the future evolution of glaciers.

Mathematicians calculate the movement of ice using complex equations, which can only be approximated by computer simulations. Glaciologists use precipitation and temperature data in order to calculate the accumulation and melting of ice. Using all information available, that of the mathematician and the glaciologist, a method for simulating the evolution of glaciers over time is constructed.

At the end of the film you can chose your own climate scenario for the 21st century and the simulate the future of the Aletsch Glacier situated in Switzerland. The possible scenarios are a temperature hike of 2°C or 4°C or a new ice age.

## Activity

- ➲ *Ice melting:* Can you calculate by how much the sea level would rise if all of the ice in Antarctica melted. Information: The ice in Antarctica covers an area of 14 million km<sup>2</sup> and is on average 2 km thick. The earth can be assumed to be a sphere of radius 6371 km and is filled up to 70% with water. Water is denser than ice, 1 m<sup>3</sup> ice are the equivalent of 0.9 m<sup>3</sup> water.



30 Top: »Mollweide« projection.  
Bottom: »Azimuthal Equidistant« and »Gnomonic« projection

# The Sphere of the Earth



This program explores the science of cartography and geometry of the earth. The geometric properties of the sphere and the plane are essentially different, as the Mathematician Carl Friedrich Gauß already proved: there is no map that faithfully represents the surface of the Earth without distortion.

With this program it is possible to study these distortions by comparing six different map projections with different spatial properties. »The Sphere of the Earth« is part of a larger exhibit including posters, a globe, a flexible ruler, and other tools and activities.

In the upper part of the screen you can select the different map projections. When clicking on the map, a so-called Tissot's ellipse is shown. A Tissot's ellipse represents a real circle on the globe that is distorted due to the projection. A red ellipse indicates a distortion, but occasionally the ellipse's boundary or interior turns green, meaning that the shape or the area of the ellipse (respectively) are not distorted at this point.

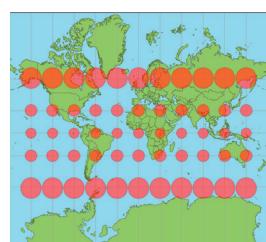
## Can we measure distortion?

A map of the world is very different from a globe. Some continents appear larger, others smaller; even their shape does not always match. Why is that? From a mathematical point of view these distortions originate in the different geometry of the map and the globe. The sphere of the earth is curved while a map is flat. This curvature of the globe means that the laws of geometry that we know from planar geometry do no longer apply here! The angular sum of the triangle is greater than  $180^\circ$ , the area of a circle is smaller than  $\pi \cdot r^2$  and so on.

Thus it is not always possible to project a triangle one-to-one from a globe to a plane. Whatever you try, you either have to distort the angles of the triangle or the distance between the corners (or both). Likewise you have to distort either the radius or the area of a circle (or both).

As a consequence there is no such thing as a perfect map: the surface of the earth can never be projected on a map without distortions. When creating a map you always have to decide on the type of projection, i.e. on how you display the proportions of the globe on the map. Many different types of projections were developed over the years, each with their own way of distorting angle and distance. To measure this distortion, a circle with arbitrary small radius around a set point is compared to its distorted counterpart in the projection. This image is called the Tissot's Indicatrix or ellipse of distortion. The more the ellipse differs from the original circle, the more distorted the map is in that point.

- These ellipses show every characteristic of a projection:
- Is the projection conformal (it preserves all angles), all ellipses are circles.
  - Is the projection true to the area, all ellipses are of equal area.
  - Is the projection equidistant, all ellipses have the same semi-axis in the direction of their length-preservation. (However, most projections are only equidistant along their circle of latitude or meridian.)



Example: Tissot's ellipses of the conformal Mercator projection

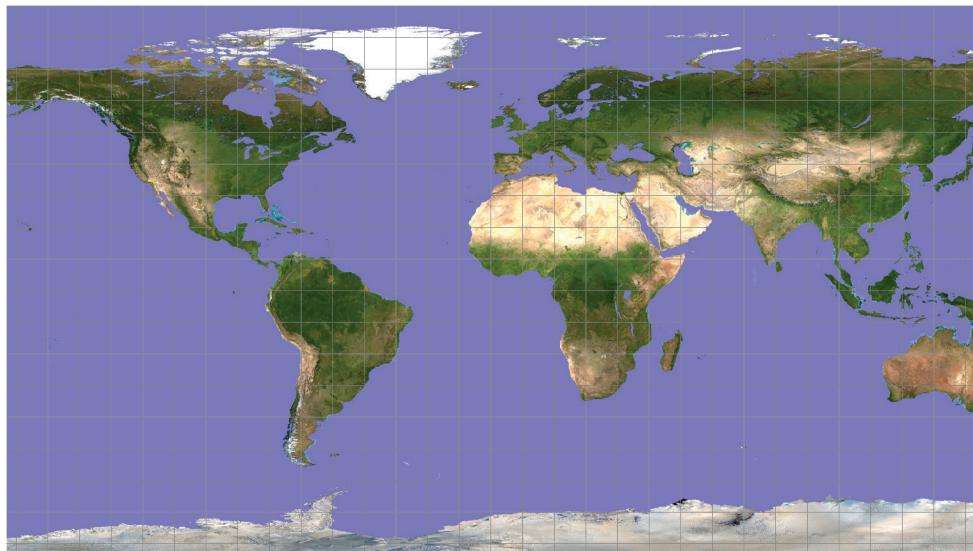


## References

- You can find additional information on the different projections and other activities on the IMAGINARY web page. There are also posters of the different maps. The movie »Dimensions« is a very good introduction into the stereographic projection of a sphere to the plane and a nice addition (chapter 1 of the movie, see the film exhibit).
- This exhibit as well as »Dune Ash« and »The Future of Glaciers« are the three winners of the »Mathematics of Planet Earth 2013« competition. You can find more information about this initiative at [www.mathofplanetearth.org](http://www.mathofplanetearth.org).
- Book recommendation: »Geometry and the Imagination« by David Hilbert and Stephen Cohn-Vossen, American Mathematical Society (October 1, 1999).

## Activities

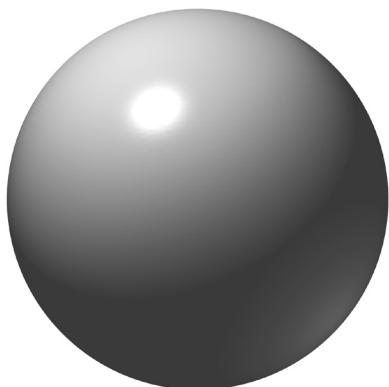
- ☒ **Size comparison:** Compare area and circumference of the continents to their image in the projections. What do you notice?
- ☒ **Projections:** Which of the projections shown are conformal/equal of area/equidistant?
- ☒ **More projections:** On the IMAGINARY web page you can also find the book »An Album of Map Projections« with more than 100 additional projections that could all be included in the program in the future



The »Plate Carée« projection

## 3D Printing

The new technology of 3D printing is a booming field – and not just for prototypes in the industry. 3D printers also have a place at home and are used in art and design. For the compilation of the print data some heavy mathematics is required.



## How are mathematical sculptures created?

Although each 3D printer builds models upwards and in layers, they differ greatly in the technique used. So called Laser-Sintering methods involve coating a rectangular even plate with a very thin layer (e.g. 0.1 mm) of plastic powder that is then fused into small beads with a laser on the points where the object is supposed to form. The plate is then lowered and the next layer applied in the same manner. Again the laser melts these points that later form the object so that they fuse with the adjoining plastic on the new layer and the layer below.

The process is repeated until the object is completed – often taking as long as several hours. Lastly, the parts not fused by the laser are extracted. This, however, means that completely closed forms are not possible to print. A hollow sphere for example cannot be made because the plastic on the inside cannot be removed.

Unfortunately, 3D printers still do not understand mathematics. But then how do you convert the mathematical description of an object into a language the printer can understand?

3D printers can only produce objects that can be enclosed by (usually small) triangles. This has established itself as the common way of describing forms. Therefore, if you want to print a large sphere in 3D, you basically give the printer a very long list of tiny triangles that as a whole form an object very similar to a sphere. If the triangles are small enough the printer does not recognize the difference between the resulting form and a sphere.

Additionally, many mathematical objects are infinite, such as a plane. To produce them as a sculpture, only a small part of the object – for example that inside a bounding sphere – can be shown. In the case of the plane and bounding sphere, the printed object would be a circular disk.

More complex mathematical objects can be produced in a similar manner. However, they do occasionally have to be thickened to make sure they do not break on the more fragile points. An example would be the Barth Sextik (see the Sculpture Gallery). There are many points in this form where two surfaces only meet in a single point. These points have to be modified for a production as a 3D printing model.



Top: IMAGINARY exhibition at the Leibniz Universität Hannover  
Bottom: SURFER film workshop for children, Vienna, school exhibition at the Otto-Hahn-Gymnasium in Saarbrücken and talk at the Girls' Day, TU Berlin

# How do you organize a Mathematics Exhibition?



Here is a short description of the main ingredients to organize your own mathematics exhibition — from the initial idea, to planning, financial matters, implementation, media communication, and hosting. An IMAGINARY exhibition can be suitably hosted at occasions such as an open-day, a science-event or an institution anniversary. You can find pictures and details on many different IMAGINARY exhibitions at [www.imaginary.org/events](http://www.imaginary.org/events).

It all begins with the initial motivation for hosting — and there is always at least one person behind it. Ideally, there are several of you in a team in order to share tasks.

The first step is to find a venue and fix a date for the exhibition. You will need it for further planning and finding sponsorship. There are many possible venues: schools, universities, banks, galleries, museums, churches, public spaces or supermarkets. The location greatly influences your public and the atmosphere of the exhibition and possibilities should be carefully evaluated. Often, the venue provides a first sponsor: the owner of the location.

Once you have secured a venue, different elements of the exhibition can be properly planned: the size, type of exhibits, the use of space, the target audience etc. We often talk of four different types of exhibits: interactive displays (for instance a program with a touch screen), hands-on displays (such as wooden puzzle or a double pendulum), movies and still images. You will find these types of displays on the IMAGINARY platform in the menu, along with many examples you can freely use.

Before putting together the displays (printing, building, preparing - don't forget the possibility of loans from existing exhibitions), you should determine your budget and look for sponsors. You will see that you can gather enthusiasm for mathematics even with little financial means.

Once you have determined your budget, you can begin contacting the media. Your links to the media are immensely important for the success of the exhibition. You can find templates for flyers and posters on the IMAGINARY platform, which you can easily adapt to fit your event.

The different elements of the exhibition now have to be prepared and put in place. If you are using exhibit items involving computers, we can only recommend to plan in sufficient time in order to test everything ahead of the opening. During the exhibition, it is important that visitors are appropriately welcomed and taken care of, if possible with the help of tour guides who are enthusiastic and can also explain the mathematics underlying the exhibits. Mathematics is often best understood through personal explanation.

When the exhibition is over, it is important to thank everyone involved and to write a report for partners and sponsors. It is worth spending some thought on what will happen with the exhibits. Perhaps they can be displayed in an art gallery, or you can enquire at your local science museum: they might even consider a permanent exhibition!

## References

- ≡ Find all data to reproduce the exhibits of the two exhibitions »IMAGINARY – through the eyes of mathematics« and »Mathematics of planet earth« at:  
[www.imaginary.org/exhibitions](http://www.imaginary.org/exhibitions)
- ≡ In the texts/background-material section of the IMAGINARY platform there is an extended manual on »How to make an IMAGINARY exhibition«
- ≡ Please contact us at [info@imaginary.org](mailto:info@imaginary.org) for more information on how to organize your own mathematics exhibition





The National Institute for Mathematical Sciences in Daejeon

The National Institute for Mathematical Sciences (NIMS) was founded in 2005 and developed over the years into a major mathematical institute in South Korea.

# The National Institute for Mathematical Sciences and the Center for Applications of Mathematical Principles

Based on fundamental research, the National Institute for Mathematical Sciences (NIMS) is trying to develop mathematical solutions for the application areas; image processing based on computational mathematics, optimizations and special-purposed cryptographic, numerical simulations and extreme-scale scientific computing, etc. NIMS also runs an active visitor program for research exchanges.

*»The era of hardware, when the infrastructure mainly determines the national competitiveness, has ended. The key to the national competitiveness is mathematics to find a way to optimize the social asset based on mathematics.«*

- Prof. Dr. Dongsu Kim, the Director of NIMS

CAMP (Center for Applications of Mathematical Principles) is a center of NIMS that aims to contribute to the development of modern mathematical research. To this end, CAMP strategically puts its focus on research exchanges and promotes interaction with industry and the public.

*Mathematics in Industry:* CAMP is actively pursuing interaction with industry. To reinforce communication and collaboration between industry and mathematics, CAMP runs industrial problem seminars, industrial postdoc programs, workshops on mathematics in industry, and summer schools on math modeling.

*Academic Programs in 2014:* In 2014 alone, CAMP hosts/supports 30 international workshops, seasonal schools and 1 thematic program. CAMP welcomes mathematicians (from postdoctoral to senior researchers) who want to join these academic programs for the periods ranging from a few days to one year.



The Mathematisches Forschungsinstitut Oberwolfach in the Black Forest

The Mathematisches Forschungsinstitut Oberwolfach (MFO) is one of the most renowned institutes of its kind. It hosts every year about 2500 mathematicians from all over the world, who use it as a convention and research center. It is appreciated in the scientific community as an institution setting high standards for its workshop and research program.

# The Mathematisches Forschungsinstitut Oberwolfach and the MiMa

Even though most mathematical research is published through electronic media, the personal contact between researchers retains its importance. This is particularly true for mathematics, where the high degree of abstraction in the subject enhances the central role of ideas exchange and personal communication. The institute invites internationally renowned experts, together with more junior scientific talents, to work together for a short, intensive time. The remote location of the institute offers ideal conditions for research activities meant to influence and stimulate the future development of a research field.

The scientific programs of the MFO are designed to encompass the entire breadth of mathematics, including its scientific and technical applications. The importance of modern mathematics for today's society is often underestimated, since it is often hiding behind the many technological and societal applications. It is astounding how much mathematical know-how from fields such as number theory, graph theory or optimization has been applied in everyday things such as mobile phone, credit cards and cars.

The MFO is also partner of a unique museum: the museum for minerals and mathematics (MiMa) in Oberwolfach. The MiMa invites you to explore the wonderful world of mathematics and minerals. Not only does it host unique mineral treasures of the area, or provide artistic views into mathematics, but it shows the captivating link between the two areas. Installations on the themes of symmetry and crystallography will give you new insights into these two fields.

## Credits and References

On the following pages you will find the credits for the NIMS-IMAGINARY exhibition and all exhibits, and links to download the the programs, 3D data, images or films.

---

### Exhibition

---

#### NIMS-IMAGINARY

Hyungju Park (Chair)  
Wonju Jeon (Administrative Coordinator)  
Chulmin Kang (Installation Manager)  
Yeon Eung Kim (Coordinator)  
Yongtak Jin (Translation, Internal Technical Support)

Seungjai Lee, Seok Hyeong Lee, Hyeji Kim,  
Dong Sung Yoon, Soojung Kim, Yoonbok  
Lee, Jaeho Shin, Sang-Gu Lee (Support,  
Translation)

---

#### MFO IMAGINARY

Andreas Daniel Matt (Project Management,  
Exhibits)  
Christian Stussak (Software, Technical  
Support)  
Bianca Violet (Communication, Exhibits)  
Christoph Knoth, Konrad Renner (Design,  
Exhibits)  
Daniel Ramos (Networking, Support)  
David Grünberg (Networking, Support)  
Carla Cederbaum (Networking)

Alejandro Baranek, Friedrich Klaus, Se-  
verina Klaus, Stephan Klaus, Antonia Mey,  
Steffen Weissmann (Support)

Gert-Martin Greuel (Scientific Advisor  
IMAGINARY)  
Gerhard Huisken (Director MFO/IMAGI-  
NARY)

---

#### Organizer

National Institute for Mathematical Sciences  
(NIMS)

---

#### Partners

International Congress of Mathematicians  
(ICM)  
Mathematisches Forschungsinstitut Oberwol-  
fach (MFO)

Booklet, texts, layout and translations

Authors of the exhibits  
David Grünberg  
Severina Klaus  
Stephan Klaus  
Oliver Labs  
Andreas Daniel Matt  
Antonia Mey  
Konrad Renner  
Susanne Schimpf

---

#### Acknowledgment:

Carla Cederbaum, Annette Disch,  
Gert-Martin Greuel, Guillaume Jouvet,  
Christoph Knoth, Lea Renner, Daniel Weiss

---

#### Exhibits

---

#### Program SURFER

Developed for IMAGINARY by the Math-  
ematisches Forschungsinstitut Oberwolfach  
in collaboration with TU Kaiserslautern and  
Martin Luther University Halle-Wittenberg.

Gert-Martin Greuel (Direction), Christian  
Stussak (Software Development, Java-Ren-  
derer), Maik Urbanek (JavaFX), Andreas  
Daniel Matt (Concept, Coordination), Anna  
Hartkopf (Coordination), Christoph Knoth  
(Design), Oliver Labs (Advice, Galleries),  
Herwig Hauser (Galleries), Maria Alberich,  
Jordi Buendia, Capi Corrales, Anna Hart-  
kopf, Herwig Hauser, Oliver Labs, Andreas  
Daniel Matt, Lara May, Anna Sabater and  
Emilio Sánchez (Texts).

#### License:

Apache 2.0 ([www.spdx.org/licenses/](http://www.spdx.org/licenses/)  
Apache-2.0)

#### Link:

[www.imaginary.org/program/surfer](http://www.imaginary.org/program/surfer)

---

### Morenements

Developed by Martin von Gagern. Picture author: Dmharvey (Wikimedia Commons).

#### License:

GPL-2.0 ([www.spdx.org/licenses/GPL-2.0](http://www.spdx.org/licenses/GPL-2.0))

#### Links:

[www.morenements.de](http://www.morenements.de)  
[www.science-to-touch.com/iOrnament](http://www.science-to-touch.com/iOrnament)  
[www.imaginary.org/program/morenements](http://www.imaginary.org/program/morenements)

---

### Cinderella

This collection of experiments was designed and developed by Jürgen Richter-Gebert and is created with Cinderella. Cinderella is a dynamic geometry software by Jürgen Richter-Gebert and Ulrich Kortenkamp.

#### License (Cinderella-Applets):

CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/CC-BY-NC-SA-3.0](http://www.spdx.org/licenses/CC-BY-NC-SA-3.0))

#### Links:

[www.cinderella.de](http://www.cinderella.de)  
[www.mathe-vital.de](http://www.mathe-vital.de)  
[www.imaginary.org/program/cinderella-applets](http://www.imaginary.org/program/cinderella-applets)

---

### Crystal Flight

Crystal Flight is a variant of Curved Spaces and part of [www.geometrygames.org](http://www.geometrygames.org). Designed and developed by Jeff Weeks.

#### License:

CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/CC-BY-NC-SA-3.0](http://www.spdx.org/licenses/CC-BY-NC-SA-3.0))

#### Links:

[www.geometrygames.org](http://www.geometrygames.org)  
[www.imaginary.org/program/crystal-flight](http://www.imaginary.org/program/crystal-flight)

---

### FroZenLight

Designed and developed by Zoltan Palmer.

#### License:

GPL-3.0 ([www.spdx.org/licenses/GPL-3.0](http://www.spdx.org/licenses/GPL-3.0))

#### Links:

[www.frozenlight.altervista.org](http://www.frozenlight.altervista.org)  
[www.imaginary.org/program/frozenlight](http://www.imaginary.org/program/frozenlight)

---

### jReality

This jReality collection was designed and developed by Ulrich Pinkall and Steffen Weissmann. The framework jReality was developed at the MATHEON, Berlin. Supported by MATHEON and TU Berlin

---

### License:

FreeBSD ([www.spdx.org/licenses/BSD-2-Clause-FreeBSD](http://www.spdx.org/licenses/BSD-2-Clause-FreeBSD))

#### Links:

[www.jreality.de](http://www.jreality.de)  
[www.imaginary.org/program/jreality-exhibit](http://www.imaginary.org/program/jreality-exhibit)

---

### Qi

Nicolas Schmitt (Developer, Mathematical Theory), members of the GeometrieWerkstatt (Mathematical Theory). Qi is a spin-off program of the visualization environment XLab, developed at the GeometrieWerkstatt, Eberhard Karls Universität Tübingen.

#### License:

CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/CC-BY-NC-SA-3.0](http://www.spdx.org/licenses/CC-BY-NC-SA-3.0))

#### Links:

[www.math.uni-tuebingen.de/ab/GeometrieWerkstatt/gallery/index.html](http://www.math.uni-tuebingen.de/ab/GeometrieWerkstatt/gallery/index.html)  
<http://www.imaginary.org/program/qi>

---

### Dune Ash (MPE)

Dietmar Kröner, Martin Nolte, Theresa Strauch, Tobias Malkmus (Organisation), M.Nolte, R. Klöfkorn, D. Nies, J. Gerstenberger, T. Malkmus, A. Pfeiffer (Programming). Images: San Jose (Wikimedia Commons), H. Thorburn (Wikimedia Commons), J. Gerstenberger. Developed at the University of Freiburg, Department of Applied Mathematics. The exhibits marked with MPE are part of the Mathematics of Planet Earth initiative and exhibition ([www.mathsofplanetearth.org](http://www.mathsofplanetearth.org)).

#### Licenses:

Software: GNU-GPL-2 ([www.spdx.org/licenses/GPL-2.0](http://www.spdx.org/licenses/GPL-2.0)), images and data: CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/CC-BY-NC-SA-3.0](http://www.spdx.org/licenses/CC-BY-NC-SA-3.0))

#### Links:

[www.dune-project.org](http://www.dune-project.org)  
[dune.mathematik.uni-freiburg.de/dune-ash](http://dune.mathematik.uni-freiburg.de/dune-ash)  
[www.imaginary.org/program/dune-ash](http://www.imaginary.org/program/dune-ash)

---

### TsunaMath (MPE)

Raouf Hamouda, Emmanuel Audusse, Jacques Sainte-Marie (Developers). Project Numerical Analysis for Geophysics and Environment (Organisation). Institutional support by CEREMA - Compiègne Research Center, UPMC - University Pierre et Marie Curie (Paris), INRIA - Rocquencourt Research Center, France.

**License:**  
CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/](http://www.spdx.org/licenses/)  
CC-BY-NC-SA-3.0)

**Links:**  
[team.inria.fr/ange/software/](http://team.inria.fr/ange/software/)

---

The Future of Glaciers (MPE)  
Guillaume Jouvet (Film, Author), Chantal Landry, Antonia Mey (Support). Glacier simulation: Guillaume Jouvet, Marco Piccasso, Jacques Rappaz, Mathias Huss, Heinz Blatter, Martin Funk. Images: CWM GmbH and VAW ETH. Created at the Department of Mathematics, Freie Universität Berlin and supported by Deutsche Forschungsgemeinschaft (Projekt KL 1806 5-1).

**License:**  
CC-BY-NC-ND-3.0 ([www.spdx.org/licenses/](http://www.spdx.org/licenses/)  
CC-BY-NC-ND-3.0)

**Links:**  
[page.mi.fu-berlin.de/jouvet/](http://page.mi.fu-berlin.de/jouvet/)  
[www.imaginary.org/film/the-future-of-glaciers](http://www.imaginary.org/film/the-future-of-glaciers)

---

The Sphere of the Earth (MPE)  
Designed and developed by Daniel Ramos of MMACA (Museu de Matemàtiques de Catalunya).

**License:**  
CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/](http://www.spdx.org/licenses/)  
CC-BY-NC-SA-3.0)

**Links:**  
[www.mmaca.cat](http://www.mmaca.cat)  
[www.imaginary.org/program/the-sphere-of-the-earth](http://www.imaginary.org/program/the-sphere-of-the-earth)

---

Sculptures  
GeometrieWerkstatt, Herwig Hauser, Wjatscheslaw Kewlin, Oliver Labs (MO-Labs.com), Nicholas Schmitt (Authors, 3D data).  
The 3D data of some algebraic surfaces is based on data originally prepared by the FORWISS Institute, University Passau.

**License:**  
CC-BY-NC-SA-3.0 ([www.spdx.org/licenses/](http://www.spdx.org/licenses/)  
CC-BY-NC-SA-3.0)

**Links:**  
[www.MO-labs.com](http://www.MO-labs.com)  
[www.imaginary.org/hands-on/four-math-sculptures](http://www.imaginary.org/hands-on/four-math-sculptures)  
[www.imaginary.org/hands-on/lawson-minimal-surface-of-genus-2](http://www.imaginary.org/hands-on/lawson-minimal-surface-of-genus-2)  
[www.shapeways.com/shops/imaginary](http://www.shapeways.com/shops/imaginary)

---

Image gallery (printed)  
Aurélien Alvarez, Luc Benard, Uli Gähnshirt, Étienne Ghys, Herwig Hauser, Tim Hoffmann, Jos Leys, Richard Palais, Ulrich Pinkall, Bianca Violet.

---

Image gallery (digital)  
Brunda Alagarsamy, Ibrahim Cahit Arkut, Luc Benard, Jean Constant, Xavier de Clippelir, Francesco de Comité, Eleonore Faber, Christian Gaier, Herwig Hauser, Sebastian Heller, Oliver Labs, Ulrich Pinkall, Ken Rubenstein, Torolf Sauermann, Nicholas Schmitt, Bogdan Soban.

Licenses and Links:  
Please refer to each individual picture at [www.imaginary.org/galleries](http://www.imaginary.org/galleries)

---

Film gallery  
Aurélien Alvarez, Aubin Arroyo, Meret Becker, Pavel Boytchev, Ester Dalvit, Diana Davis, Victor Fernandes, Rudi Friemel, Pierre-Olivier Gaumin, Étienne Ghys, Steven Higley, Hans Katzgraber, Stephan Klaus, Jos Leys, Christian Magnes, Andreas Matt, Sebastian Minjeaud, Armindo Moreira, Maëlle Nodef, Renate Quehenberger, José Francisco Rodrigues, Antoine Rousseau, Torsten Stier, Kathrin Stumreich, Nikola Tasic, Ulliroyal, Bianca Violet.

Licenses and Links:  
Please refer to each individual film at [www.imaginary.org/films](http://www.imaginary.org/films)





Mathematisches  
Forschungsinstitut  
Oberwolfach

Klaus Tschira Stiftung  
gemeinnützige GmbH

